Inflation, output growth and their uncertainties in South Africa: Empirical evidence from an asymmetric multivariate GARCH-M model

The study examines the relationships between inflation uncertainty and output growth uncertainty, and analyzes their effects on the level of inflation and output growth in South Africa. An asymmetric multivariate GARCH-M model suggested by Grier et al. (2004) is applied. The findings suggest that while uncertainty about growth is detrimental to output growth, inflation uncertainty is not. The findings further reveal that Cukierman and Meltzer (1986) hypothesis of a positive impact of inflation uncertainty on the level of inflation is supported, and there exists a negative impact of output growth uncertainty on inflation. No association could be found between inflation uncertainty and output growth uncertainty. The findings imply that output growth and its uncertainty should not be treated separately as it is usually suggested by business cycle models. In addition, both output growth uncertainty and inflation uncertainty should be considered as part of the determinants of output growth in South Africa.

Keywords: inflation; growth; uncertainty; asymmetric multivariate GARCH-M model; South Africa.

JEL classification: E31; E52; C32.

1. Introduction

In his Nobel Prize lecture, Friedman (1977) pointed out that greater inflation is associated with greater inflation uncertainty and that inflation uncertainty is harmful to growth since it creates economic inefficiency through distortions in the effectiveness of the price mechanism thus hindering the efficient allocation of resources. Although Ball (1992) supports his idea on the link between inflation and its uncertainty, Cukierman and Meltzer (1986) and Holland (1995) explored the possibility of a reverse causality. Cukierman and Meltzer (1986) argue that an increase in inflation uncertainty leads to an increase in the level of inflation as policymakers create surprise inflation to stimulate output. Holland (1995) in his stabilization hypothesis points out that an increase in inflation uncertainty leads to a decrease in inflation as policymakers reduce the growth rate of money (hence lowering inflation) to attenuate the effects of inflation uncertainty on the economy.

Considering the impact of inflation uncertainty on growth, contrary to Friedman (1977), Dotsey and Sarte (2000) suggest a positive impact of inflation uncertainty on output growth. They argue that an increase in the variability of monetary growth, and therefore inflation, makes...
the return to money balances more uncertain and leads to a fall in the demand for real money balances and consumption. Hence, agents increase precautionary savings leading to an increase in investments which in turn boost output growth.

The impact of real uncertainty (output growth uncertainty) on output growth has also attracted considerable debate among scholars. While Friedman (1968) argues that there is an independence relationship between the two, Black (1987) and Blackburn (1999) suggest a positive impact of output growth uncertainty on output growth by arguing that high volatility will lead to an increase in savings through precautionary motives which in turn result in an increase in investments. Pindyck (1991) and Ramey and Ramey (1991) on the other hand suggest a negative impact of output growth uncertainty on output growth. According to them, large fluctuations in economic activity are more likely to increase the uncertainty regarding the long-run profitability of investments, making the returns of investments riskier and hence reducing the level of investments and therefore output growth.

Scholars have also examined the relationship between inflation uncertainty and output growth uncertainty. Taylor (1981) and Fuhrer (1997) suggest a trade-off between inflation uncertainty and output growth uncertainty. According to them, the objective of stabilizing inflation can be achieved only at the expense of accepting a high volatile output growth and vice-versa. In contrast, Logue and Sweeney (1981) suggest a positive link from inflation volatility to output growth volatility in contrast to Devereux (1989) who supports a positive link from output growth volatility to inflation volatility.

A number of empirical studies have sought to examine the relationship between inflation uncertainty and output growth uncertainty and to analyze their effects on the levels of inflation and output growth, using either one-step approach in GARCH-in-mean models (see, for instance, (Grier et al., 2004; Grier, Grier, 2006; Olayinka, Hassan, 2010; Ndou, Mokoena, 2011; Omay, 2011; Hachicha, Hooi Hooi, 2013)) or two-step approach where uncertainty measures for inflation and output growth are initially generated using various types of GARCH models and then causality tests are conducted to examine the links between the variables (see for instance, (Fountas, Karanasos, 2007; Thornton, 2007; Korap, 2009; Mehrara, Mojab, 2010; Narayan, Narayan, 2013)).

However, to generate uncertainty measures of inflation and output growth, a number of studies use univariate models (see, for instance, (Fountas, Karanasos, 2007; Ndou, Mokoena, 2011; Hachicha, Hooi Hooi, 2013; Farhan et al., 2012; Mohd et al., 2012)), restrictive models of the covariance process such as CCC and DCC-GARCH² models or diagonal BEKK model (see for instance, (Fountas et al., 2002; Jiranyakul, Opiela, 2011; Korap, 2009; Mehrara, Mojab, 2010; Türkyılmaz, Ozer, 2010; Conrad, Karanasos, 2008)). As Grier et al. (2004) note, univariate models do not permit the joint generation of the uncertainty measures for inflation and output growth nor do they permit one to simultaneously examine the relationships between inflation, growth and their uncertainties while on the other hand, the restrictive models can lead to misspecification problem.

In addition, when generating uncertainty measures through GARCH models, Grier et al. (2004) cautioned that imposing diagonality and symmetry restrictions on the variance-covariance matrix of output growth and inflation might lead to misspecification hence wrong uncertainty

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² CCC and DCC-GARCH models are respectively Constant and Dynamic Conditional Correlation GARCH model of Bollerslev (1990) and Engle (2002). They both assume that the matrices of ARCH and GARCH terms are diagonal.
measures and faulty conclusions regarding the links between inflation, output growth and their uncertainties. To address this problem, Grier et al. (2004) proposed a model which allows testing for diagonality and symmetry in the variance-covariance matrix instead of imposing them.

This study therefore follows Grier et al. (2004) and applies an asymmetric multivariate GARCH-M model to examine the relationships between inflation uncertainty and output growth uncertainty and to analyze their effects on the level of inflation and output growth in South Africa.

The findings of this study suggest that in South Africa, while uncertainty about growth is detrimental to output growth, inflation uncertainty is not. Inflation uncertainty promotes output growth as suggested by Dotsey and Sarte (2000). Friedman’s (1977) hypothesis of a negative impact of inflation uncertainty on output growth was not supported in this study. The findings further reveal that Cukierman and Meltzer (1986) hypothesis of a positive impact of inflation uncertainty on the level of inflation is supported, and there exists a negative impact of output growth uncertainty on inflation. The results however indicate that there is no association between inflation uncertainty and output growth uncertainty.

The rest of the paper is organized as follows. Section 2 highlights the methodology used. Section 3 presents and interprets the estimation results and section 4 concludes the study.

2. Methodology

To examine the relationship between inflation uncertainty and output growth uncertainty and further analyze their effects on the levels of inflation and output growth in South Africa, this study follows (Grier et al., 2004) and uses an asymmetric BEKK3 GARCH-M model in which the conditional means of inflation ($\pi_t$) and output growth ($y_t$) are in form of VARMA (Vector AutoRegressive Moving Average) GARCH-M model, where the conditional standard deviations of output growth and inflation are included as explanatory variables in each conditional mean equation. The specification of the conditional means of inflation ($\pi_t$) and output growth ($y_t$) is as follows:

$$
Y_t = \mu + \sum_{i=1}^{p} \Gamma_i Y_{t-i} + \Psi \sqrt{h_t} + \sum_{j=1}^{q} \Theta_j \varepsilon_{t-j} + \varepsilon_t
$$

with $\varepsilon_t \mid \Omega_t \sim N(0, H_t)$, where $\Omega_t$ represents the information set available at time $t$. In addition, $E(\varepsilon_{y,t}^2) = h_{y,t}$; $E(\varepsilon_{\pi,t}^2) = h_{\pi,t}$; $E(\varepsilon_{y,t}, \varepsilon_{\pi,t}) = h_{y\pi,t}$;

$$
H_t = \begin{bmatrix} h_{y,t} & h_{y\pi,t} \\ h_{y\pi,t} & h_{\pi,t} \end{bmatrix}; \quad Y_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}; \quad \varepsilon_t = \begin{bmatrix} \varepsilon_{y,t} \\ \varepsilon_{\pi,t} \end{bmatrix}; \quad \sqrt{h_t} = \begin{bmatrix} \sqrt{h_{y,t}} \\ \sqrt{h_{\pi,t}} \end{bmatrix}; \quad \mu = \begin{bmatrix} \mu_y \\ \mu_{\pi} \end{bmatrix}; \quad \Gamma_i = \begin{bmatrix} \Gamma^{(i)}_{y} & \Gamma^{(i)}_{y\pi} \\ \Gamma^{(i)}_{y\pi} & \Gamma^{(i)}_{\pi} \end{bmatrix};
$$

$$
\Psi = \begin{bmatrix} \psi_{yy} & \psi_{y\pi} \\ \psi_{y\pi} & \psi_{\pi\pi} \end{bmatrix}; \quad \Theta_j = \begin{bmatrix} \theta^{(j)}_{yy} & \theta^{(j)}_{y\pi} \\ \theta^{(j)}_{y\pi} & \theta^{(j)}_{\pi\pi} \end{bmatrix},
$$

BEKK model is a multivariate GARCH model developed by Engle and Kroner (1995) and was named after Baba, Engle, Kraft and Kroner.

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3 BEKK model is a multivariate GARCH model developed by Engle and Kroner (1995) and was named after Baba, Engle, Kraft and Kroner.
where $H_t$ is the conditional variance-covariance matrix, $h_{yt}$ is the conditional variance of output growth, $h_{yt}^{xt}$ is the conditional variance of inflation, $h_{yt}^{xt}$ and $h_{yt}^{xt}$ are the conditional covariances between inflation and output growth, $\varepsilon_t$ is the vector of error terms, $\mu$ is the matrix of constant terms, $\Gamma_t$ is the matrix of Autoregressive coefficients, $\Psi$ is the matrix of in-mean coefficients, and $\Theta_t$ is the matrix of moving average coefficients. It is important to note that in GARCH models, uncertainty (volatility) is captured by the conditional variance which is simply the variance of the one step ahead forecasting error. The conditional variance-covariance matrix of an asymmetric BEKK model is written as:

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'H_{t-1}B + D'\omega_{t-1}\omega_{t-1}'D,$$

where $C = \begin{bmatrix} c_{yy} & 0 \\ c_{xx} & c_{xx} \end{bmatrix}$; $A = \begin{bmatrix} \alpha_{yy} & \alpha_{yx} \\ \alpha_{xy} & \alpha_{xx} \end{bmatrix}$; $B = \begin{bmatrix} \beta_{yy} & \beta_{yx} \\ \beta_{xy} & \beta_{xx} \end{bmatrix}$; $D = \begin{bmatrix} \delta_{yy} & \delta_{yx} \\ \delta_{xy} & \delta_{xx} \end{bmatrix}$; $\omega = \begin{bmatrix} \omega_{y,t} \\ \omega_{x,t} \end{bmatrix}$.

In equation (2), $C$ is a lower triangular matrix of constant terms, $A$ is a matrix of ARCH coefficients which captures the ARCH effects, and $B$ is a matrix of GARCH coefficients capturing the GARCH effects. The diagonal elements in matrix $A$ show the impact of own past shocks on the current conditional variance, the diagonal elements in Matrix $B$ represent the impact of own past volatility on the current conditional variance, while the off-diagonal elements in matrices $A$ and $B$ represent the volatility spillovers’ effects (Xu, Sun, 2010). Asymmetry in the conditional variance-covariance matrix is captured by the matrix $D$ which is the matrix of asymmetric coefficients. To introduce asymmetry in the conditional variance-covariance process, Grier et al. (2004) use the concepts of good and bad news. If inflation is higher than expected, it is bad news, captured by a positive inflation residual defined as $\omega_{yt} = \max(\varepsilon_{yt}, 0)$. Similarly, if output growth is lower than expected, it is also bad news, captured by negative output growth innovations defined as $\omega_{yt} = \min(\varepsilon_{yt}, 0)$. We note that the BEKK model becomes symmetric if asymmetric coefficients are statistically jointly equal to 0, i.e. $\delta_{ij}$, for all $i,j = y, x$. It should also be noted that BEKK model is preferred because it ensures the positive definiteness of the conditional variance-covariance matrix unlike the other variants of multivariate GARCH models.

Equation (2) can also be written as follows:

$$h_{yt} = c_{yy}^2 + c_{yx}^2 + c_{xx}^2 + 2\alpha_{yy}\alpha_{yx}\varepsilon_{yt-1}\varepsilon_{yt-1} + 2\alpha_{yy}\alpha_{xx}\varepsilon_{yt-1}\varepsilon_{yt-1} + 2\alpha_{yy}\alpha_{xx}\varepsilon_{yt-1}\varepsilon_{yt-1} + \beta_{yy}^2 h_{yt-1} + 2\beta_{yx}\beta_{yx} h_{yt-1} + 2\beta_{xx}^2 h_{yt-1} +$$

$$+ \delta_{yy}^2 \omega_{yt-1}^2 + 2\delta_{yx}\delta_{yx}\omega_{yt-1} \omega_{yt-1} + \delta_{xx}^2 \omega_{yt-1}^2,$$

$$h_{xt} = c_{xt}^2 + c_{xy}^2 + c_{xx}^2 + 2\alpha_{xy}\alpha_{yx}\varepsilon_{yt-1}\varepsilon_{yt-1} + 2\alpha_{xy}\alpha_{xx}\varepsilon_{yt-1}\varepsilon_{yt-1} + 2\alpha_{xy}\alpha_{xx}\varepsilon_{yt-1}\varepsilon_{yt-1} + \beta_{xy}^2 h_{yt-1} + 2\beta_{yx}\beta_{yx} h_{yt-1} + 2\beta_{xx}^2 h_{yt-1} +$$

$$+ \delta_{xy}^2 \omega_{yt-1}^2 + 2\delta_{yx}\delta_{yx}\omega_{yt-1} \omega_{yt-1} + \delta_{xx}^2 \omega_{yt-1}^2.$$

From the conditional mean equation (1), we can check how inflation uncertainty and output growth uncertainty affect the level of inflation and output growth. Assessing the impact of output growth uncertainty and inflation uncertainty on output growth is done by respectively testing...
the null hypotheses that $\psi_{yy} = 0$ and $\psi_{yp} = 0$. A positive $\psi_{yy}$ would mean a positive impact of output growth uncertainty on output growth, which is Black hypothesis while a negative $\psi_{yy}$ would imply a negative impact of output growth uncertainty on output growth, supporting the views of Pindyck (1991) and Ramey and Ramey (1991). A positive $\psi_{yp}$ would mean a positive impact of inflation uncertainty on output growth which is Dotsey–Sarte hypothesis while a negative $\psi_{yp}$ would mean a negative impact of inflation uncertainty on output growth, which is Friedman hypothesis.

Similarly, testing the impact of output growth uncertainty and inflation uncertainty on the level of inflation is done by respectively testing whether $\psi_{py} = 0$ and $\psi_{yp} = 0$. A positive $\psi_{py}$ would mean a positive impact of output growth uncertainty on inflation which is referred to as the Devereux hypothesis while a negative $\psi_{py}$ would imply a negative impact of output growth uncertainty on inflation. On the other hand, a positive $\psi_{yp}$ would mean a positive impact of inflation uncertainty on inflation, which would support Cukierman–Meltzer hypothesis while a negative $\psi_{yp}$ would mean a negative impact of inflation uncertainty on inflation, which is the stabilization hypothesis of Holland (1995).

From the conditional variance-covariance matrix (equation (3)), it can be seen that the impact of the conditional variance of one variable on the conditional variance of another variable is captured by the off-diagonal elements of the matrix $B$, $\beta_{xy}$ and $\beta_{yx}$. Examining the impact of inflation uncertainty on output growth uncertainty and vice versa is done respectively by testing $\beta_{yx} = 0$ and $\beta_{xy} = 0$. A negative $\beta_{yx}$ or $\beta_{xy}$ would support a trade-off hypothesis between inflation uncertainty and output growth uncertainty suggested by Taylor (1981). A positive $\beta_{yx}$ would support Logue and Sweeney (1981) hypothesis while a positive $\beta_{xy}$ would support Devereux (1989) hypothesis.

3. Empirical results and discussion

Monthly data on price and output levels for South Africa are used for the period February 1961 to March 2012. Data were retrieved from International Financial Statistics (IFS) of the International Monetary Fund (IMF). The price level is captured by Consumer Price Index (CPI) and output level is captured by Manufacturing Production Index. This is because high frequency data which are more appropriate with GARCH models are not available for GDP for most countries including South Africa; proxies for output level such as industrial production index, manufacturing production index, crude oil production index, etc. are therefore commonly used in this kind of studies (see, for instance, (Korap, 2009; Türkyılmaz, Özer, 2010; Olayinka, Hassan, 2010; Jiranyakul, Opiela, 2011; Bipradas, 2012; Hachicha, Hooi Hooi, 2013)).

Inflation rate is computed as the monthly difference of the logarithm of CPI,

$$\pi_t = [\log(CPI_t / CPI_{t-1})] \times 100,$$

and output growth is computed as the monthly difference of the logarithm of the production index ($Y_t$), $y_t = [\log(Y_t / Y_{t-1})] \times 100$.

Summary statistics in Panel A of Table 1 show that both inflation, $\pi$, and output growth, $y$, are positively skewed and display platikurtic behavior. In addition, Jarque–Bera (1987) test rejects the null hypothesis of normality in both inflation and output growth series.
Unit root tests, serial correlation and ARCH tests are preliminarily conducted. Unit root tests are conducted in order to assess the order of integration of the series, and ARCH test, to check for evidence of conditional heteroscedasticity in the data, that is, whether the variances of the series are time-varying. As Grier and Perry (1998) points out, one should be able to reject the null hypothesis of constant variance before estimating a GARCH model and generate uncertainty measures. An endogenous two-break unit root test of Lee and Strazicich (2003) and a non-parametric unit root test of Breitung (2002) are used to test for unit root in the series. Serial correlation is tested using the Ljung–Box (1978) test on the series and on the squared series, while testing for the presence of ARCH effects in the series is done using LM-ARCH test of Engle (1982).

![Inflation](image1)

![Output Growth](image2)

**Fig. 1.** Inflation and output growth in South Africa (1961–2012)

Unit root test results reported in Panel B of Table 1 indicate that both tests strongly reject the null hypothesis of a unit root in inflation and output growth series for South Africa. Inflation and output growth series are hence integrated of order 0, $I(0)$. This implies that there is no need to difference them when estimating the Mean equations. In addition, Ljung–Box (1978) test rejects the null hypothesis of no serial correlation in both the series and squared series (Panel C, Table 1). Serial correlation in the squared data is an evidence of conditional heteroscedasticity and this is confirmed by LM-ARCH test of Engle (1982). ARCH test results in Panel C of Table 1 indeed suggest that inflation and output growth exhibit significant volatility clustering for South Africa, implying that the variances of inflation and output growth are not constant but time-varying. Figure 1 seems to confirm this.
Since the presence of ARCH effects is confirmed, we proceed to estimate our asymmetric BEKK GARCH-M\(^4\) model for South Africa. The estimation results are in Table 2 along with the diagnostic test results as well as the coefficients restriction tests on the estimated model.

**Table 1.** Preliminary tests results

*Panel A. Summary statistics*

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Variance</th>
<th>Skewness</th>
<th>Excess kurtosis</th>
<th>Jarque–Bera test</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi)</td>
<td>0.6786</td>
<td>0.4021</td>
<td>1.176 ([0.000])</td>
<td>2.863 ([0.000])</td>
<td>351.308 ([0.000])</td>
</tr>
<tr>
<td>(y)</td>
<td>0.2644</td>
<td>8.1814</td>
<td>0.215 ([0.000])</td>
<td>0.930 ([0.000])</td>
<td>26.895 ([0.000])</td>
</tr>
</tbody>
</table>

*Panel B. Unit root tests*

<table>
<thead>
<tr>
<th></th>
<th>Lee–Strazicich unit root test</th>
<th>Breitung test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\tau) Stat</td>
<td>Breaks</td>
</tr>
<tr>
<td>(\pi)</td>
<td>(-13.20***) ({2})</td>
<td>1985m12</td>
</tr>
<tr>
<td>(y)</td>
<td>(-15.83***) ({7})</td>
<td>1980m6</td>
</tr>
</tbody>
</table>

*Panel C. Univariate serial correlation and ARCH test*

<table>
<thead>
<tr>
<th></th>
<th>(Q(10))</th>
<th>(Q(20))</th>
<th>(Q^2(10))</th>
<th>(Q^2(20))</th>
<th>ARCH(20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi)</td>
<td>465.9 ([0.000])</td>
<td>894.0 ([0.000])</td>
<td>125.9 ([0.000])</td>
<td>360.0 ([0.000])</td>
<td>8.30 ([0.000])</td>
</tr>
<tr>
<td>(y)</td>
<td>369.8 ([0.000])</td>
<td>568.6 ([0.000])</td>
<td>74.81 ([0.000])</td>
<td>99.71 ([0.000])</td>
<td>4.29 ([0.000])</td>
</tr>
</tbody>
</table>

**Notes.** Lee–Strazicich (LS) test was performed using WinRATS Pro 8.1 while Breitung test was performed using EasyReg software. Serial correlation and ARCH tests were performed using OxMetrics 6.30. Between \(\{\}\) are the optimal lags used in LS test, selected using the usual criteria and between brackets \([\]\) are the \(p\)-values. For LS test, 1% C. V is \(-5.823\); 5% C. V is \(-5.286\) and 10% C. V is \(-4.989\) for the model allowing for a shift in intercept and change in trend slope. \(P\)-values reported in brackets \([\]\) for Breitung test are based on 1000 simulations.

Diagnostic tests, Ljung–Box test, and McLeod–Li test are first used to check for the adequacy of the GARCH model estimated, in other words, to check whether the conditional mean and the conditional variance-covariance equations are well specified (see Table 2, Panel C). At 5% level, Ljung–Box test indicates that there is no serial correlation of 5\(^{th}\) and 10\(^{th}\) order in the standardized residuals of inflation and output growth mean equations. Similarly, McLeod–Li test indicates that the squares of the standardized residuals of inflation and output growth equations are also serially independent at 5% level, implying that there are no remaining ARCH/GARCH effects. The conditional mean and conditional variance-covariance equations are hence well specified.

In addition, some coefficient restriction tests (see Table 2, Panel B) are conducted in order to check whether some of the coefficients in the Mean equation and in the conditional variance-covariance matrix are redundant. The results indicate that the hypotheses of Diagonal VARMA, no GARCH, no GARCH-M, no Asymmetry and Diagonal GARCH are all rejected at 1\% significance level. This suggests that none of the terms included in the conditional mean and variance-covariance equations are redundant. Coefficient restriction tests confirm that the form of the mean

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\(^4\) In estimating the mean equation, we consider \(p = q = 2\) and the diagnostic tests confirm that the mean equation is well specified with that lag order.
equation adopted (vector autoregressive moving average, VARMA, plus the in-mean coefficients included) properly captures the dynamics of inflation and output growth in South Africa and that the form of the conditional variance-covariance matrix adopted (asymmetry and non-diagonality) also adequately captures the dynamics of the conditional variance of inflation and conditional variance of output growth. The conditional standard deviations of inflation and output growth capturing inflation uncertainty and output growth uncertainty for South Africa are in Figure 2.

Figure 2 indicates that the greatest inflation uncertainty (volatility) is seen in the 2nd half of the 1970s, in the 1980s, 1994, 2000, 2004, and 2010, while the greatest output growth uncertainty (volatility) is apparent in 1977 and 2009. Some high output growth uncertainty can also be seen in the second half of the 1970s, in 1980s, 1999, and 2004. On average, inflation uncertainty seems to have been lower than output growth uncertainty.

Table 2. Estimation results of an asymmetric BEKK GARCH-M model for South Africa

Panel A. Conditional mean equations

\[ Y_t = \mu + \sum_{j=1}^{p} \Gamma_j Y_{t-j} + \Theta_j \epsilon_{t-j} + \epsilon_t, \]
where \( \epsilon_t \sim N(0, H_t) \),

\[
\begin{align*}
\mu &= \begin{bmatrix} 2.2727 \\ 0.1037 \end{bmatrix} \\
\Gamma_1 &= \begin{bmatrix} -1.0031 \\ -0.0239 \end{bmatrix} \\
\Theta_1 &= \begin{bmatrix} 0.4436 \\ 0.0453 \end{bmatrix}
\end{align*}
\]

Panel B. Conditional variance — covariance

\[ H_t = C'C + A'\epsilon_{t-1}\epsilon_{t-1}'A + B'H_{t-1}B + D'\omega_{t-1}\omega_{t-1}'D, \]

\[
\begin{align*}
C &= \begin{bmatrix} 1.5106 \\ 0.0116 \end{bmatrix} \\
A &= \begin{bmatrix} 0.2521 \\ -0.2072 \end{bmatrix} \\
B &= \begin{bmatrix} 0.038 \\ 0.279 \end{bmatrix} \\
D &= \begin{bmatrix} 0.3674 \\ 0.0116 \end{bmatrix}
\end{align*}
\]

Diagonal VARMA: \{ \[ H_0 : \Gamma_{xy} = \beta_{xy} = \delta_{xy} = 0, \quad i = 1,2; \quad \chi^2(8) = 207659.3 [0.000] \] \}

No GARCH: \{ \[ H_0 : \alpha_{y} = \beta_{y} = \delta_{y} = 0, \quad \forall i, j = x, y; \quad \chi^2(12) = 219448.3 [0.000] \] \}

No GARCH-M: \{ \[ H_0 : \psi_{y} = 0, \quad \forall i, j = x, y; \quad \chi^2(4) = 2236.9 [0.000] \] \}

No Asymmetry: \{ \[ H_0 : \delta_{y} = 0, \quad \forall i, j = x, y; \quad \chi^2(4) = 856.3 [0.000] \] \}

Diagonal GARCH: \{ \[ H_0 : \alpha_{xy} = \alpha_{yx} = \beta_{xy} = \beta_{yx} = \delta_{xy} = \delta_{yx} = 0; \quad \chi^2(6) = 81.1 [0.000] \] \}

Panel C. Diagnostic tests

<table>
<thead>
<tr>
<th></th>
<th>Ljung–Box Q (5)</th>
<th>McLeod–Li (5)</th>
<th>Ljung–Box Q (10)</th>
<th>McLeod–Li (10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_{y,t} )</td>
<td>6.4460 [0.265]</td>
<td>1.17 [0.952]</td>
<td>13.0139 [0.222]</td>
<td>6.1748 [0.800]</td>
</tr>
<tr>
<td>( z_{x,t} )</td>
<td>7.0156 [0.219]</td>
<td>2.2824 [0.808]</td>
<td>17.1519 [0.071]</td>
<td>3.7045 [0.959]</td>
</tr>
</tbody>
</table>

Notes. Results from our estimations using WinRATS Pro 8.1. Between parentheses (∙) are the standard errors and between brackets [∙] are the p-values. \( z_{y,t} \) is the standardized residual defined as \( z_{y,t} = \epsilon_{y,t}/\sqrt{H_{y,t}} \), where \( j = y, x \).
It is to be noted that rejecting the hypothesis of no GARCH confirms that the conditional variance-covariance matrix is heteroscedastic, that is, the conditional variances of inflation and output growth are time-varying. In addition, rejecting the hypotheses of symmetry and diagonality in the conditional variance-covariance matrix shows that assuming symmetry and diagonality could have led to misspecification in the conditional variance-covariance matrix.

We now focus on the objective of the study which is to examine the relationships between inflation uncertainty and output growth uncertainty and to analyze their effects on the levels of inflation and output growth in South Africa, the estimation results of the conditional mean equations in Panel A of Table 2 indicate that the null hypotheses that $\psi_{yy} = 0$ and $\psi_{yp} = 0$ are rejected respectively at 1% and 10% level (the $p$-values not reported in the results table for the two null hypotheses are respectively equal to $p = 0.000$ and $p = 0.059$). In addition, in the output growth equation, the coefficient of the conditional standard deviation of output growth (output growth uncertainty) is negative ($\psi_{yy} = -0.4359$) while the coefficient of the conditional deviation of inflation (inflation uncertainty) is positive ($\psi_{yp} = 0.8442$). The results therefore suggest a negative impact of output growth uncertainty on output growth in South Africa supporting the views of Pindyck (1991) and Ramey and Ramey (1991), and a positive impact of inflation uncertainty on output growth which supports Dotsey–Sarte hypothesis.

Considering the impact of inflation uncertainty on output growth, our findings contradict those of Ndou and Mokoena (2011) who suggest that inflation uncertainty is detrimental to output growth in South Africa. Additionally, although our findings suggest a negative impact
of output growth uncertainty on output growth, Narayan and Narayan (2013) found that the impact of output growth uncertainty on output growth is positive.

Similarly, the results suggest that the null hypotheses that \( \psi_{xy} = 0 \) and \( \psi_{xx} = 0 \) are rejected respectively at 1% and 5% level of significance (the \( p \)-values not reported in the results table for the two null hypotheses are respectively equal to \( p = 0.000 \) and \( p = 0.030 \)). The estimated coefficient of output growth uncertainty (conditional standard deviation of output growth) is negative (\( \psi_{xy} = -0.043 \)), suggesting a negative impact of output growth uncertainty on inflation. Moreover, the coefficient of inflation uncertainty (conditional standard deviation of inflation) \( \psi_{xx} \) is positive (\( \psi_{xx} = 0.0115 \)), implying a positive impact of inflation uncertainty on inflation and supporting hence Cukierman–Meltzer hypothesis. It should be noted that our findings on the impact of inflation uncertainty on inflation contradict those of Ndou and Mokoena (2011) who support the stabilization hypothesis of Holland (1995).

We note also that our findings for South Africa on the impact of output growth uncertainty on output growth differ from Narayan and Narayan (2013). While Narayan and Narayan (2013) found the impact of output growth uncertainty on output growth to be positive, our findings suggest that output growth uncertainty does not affect output growth although the causal impact is negative.

Moreover, the results in Panel B of Table 2 show that the off-diagonal elements in matrix \( B \), \( \beta_{yy} \) and \( \beta_{xx} \) are statistically insignificant since the hypotheses that \( \beta_{xy} = 0 \) and \( \beta_{yx} = 0 \) could not be rejected even at 10% level (the \( p \)-values not reported in the results table for the two null hypotheses are respectively equal to \( p = 0.986 \) and \( p = 0.169 \)). This indicates that there is no association between inflation uncertainty and output growth uncertainty in South Africa. Consequently, the trade-off hypothesis of Fuhrer (1997), Logue–Sweeney hypothesis or Devereux hypothesis could not be supported for South Africa.

However, the diagonal elements in matrix \( B \), \( \beta_{yy} \) and \( \beta_{xx} \) are statistically significant at 1% level (the \( p \)-values not reported in the results table for the null hypotheses \( \beta_{yy} = 0 \) and \( \beta_{xx} = 0 \) are both equal to \( p = 0.000 \)), implying that own past volatility (uncertainty) affect the conditional variances of inflation and output growth in South Africa. In addition, the results show that the diagonal and off-diagonal elements in matrix \( A \) are all statistically significant except for \( \alpha_{xy} \), suggesting that own past shocks affect the conditional variances of inflation and output growth for South Africa while shocks spillovers affect only the conditional variance of inflation.

Analysis of matrix \( D \) shows that all its elements are statistically significant except \( \delta_{xx} \), however, the overall coefficient matrix is significant at 1% level (the \( p \)-value for the hypothesis that \( \delta_{yy} = \delta_{yx} = \delta_{xy} = \delta_{xx} = 0 \) is equal to \( p = 0.000 \)). Since both \( \alpha_{xx} \) and \( \delta_{xx} \) are statistically significant, it shows that inflation displays own variance asymmetry implying that a positive inflation shock leads to more inflation uncertainty than a negative inflation shock of the same magnitude. Similarly, the significance of both \( \alpha_{yy} \) and \( \delta_{yy} \) indicates that output growth also displays own variance asymmetry, which implies that a negative output growth shock leads to more output growth uncertainty than a positive output growth shock of the same magnitude.

4. Concluding remarks

The study sought to examine the relationships between inflation uncertainty and output growth uncertainty and to analyze their effects on the levels of inflation and output growth in South Africa using an asymmetric BEKK GARCH-M model suggested by Grier et al. (2004).
The findings suggest that in South Africa, while uncertainty about growth is detrimental to output growth, inflation uncertainty is not. Inflation uncertainty promotes output growth as suggested by Dotsey and Sarte (2000). Friedman’s (1977) hypothesis of a negative impact of inflation uncertainty on output growth was therefore not supported in this study. The findings further reveal that Cukierman and Meltzer (1986) hypothesis of a positive impact of inflation uncertainty on the level of inflation is supported for the case of South Africa, and there exists a negative impact of output growth uncertainty on inflation. The results however indicate that there is no association between inflation uncertainty and output growth uncertainty. The evidence of the impact of output growth uncertainty and inflation uncertainty on output growth in this study implies that output growth and its uncertainty should not be treated separately as suggested by business cycle models, and that both output growth uncertainty and inflation uncertainty should be considered as part of the determinants of output growth in South Africa.

**References**


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