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Type I error of t-tests from the simple moving average technical trading rules

Numerous studies have demonstrated the inefficiency of the market via applications of t-tests on returns from Buy and Sell Days. In this study, we highlight large type I errors associated with the tests and show that it is inappropriate to use t-tests on returns from Buy and Sell Days to support market inefficiency.

Keywords: efficient market; moving average trading rules; t-tests; type I error.
JEL classification: C1; G0.

1. Introduction

Whereas fundamental analysis uses a firm's financial statements to evaluate the company's performance, technical trading rules seek to identify patterns and trend changes in market prices. As seen in studies such as (Brock et al., 1992; Chang et al., 2006; Coutts, Cheung, 2000; Kirkpatrick II, Dahlquist, 2015; Pring, 2014), researchers often focus on comparisons of mean returns from Buy and Sell Days in technical trading strategies to conclude that technical trading strategies may be profitable, thus implying market inefficiency. In this study, we present quantitative evidence that suggests that the latter conclusions are in fact spurious.

One of the most popular technical trading rules is the simple moving average technical trading rule (Pring, 2014). Under the simple moving average trading rule, a Buy (Sell) signal emits when short-term moving averages exceed (fall below) long-term moving averages.

Conditions can be added to the simple moving average trading rule by specifying different lengths of short-term and long-term averages or a band of α percentages. We will denote these conditions as $\mathrm{MA}(S, L, a)$, where S is the length of short-term moving averages, L is the length of long-term moving averages, and a is the percentage band.

As in (Coutts, Cheung, 2000), and (Kwon, Kish, 2002), a day at time t is "classified or labeled" as a

$$\text{Buy Day if } \sum_{i=t-S}^{t} P_i \geq (1+a)\sum_{i=t-L}^{t} P_i\text{; otherwise, a Sell Day if } \sum_{i=t-S}^{t} P_i < (1+a)\sum_{i=t-L}^{t} P_i. \qquad (1)$$

Following a study by Brock et al. (1992), numerous academic articles have been published using moving average trading rules and t-tests on Buy and Sell Day returns to test for market

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efficiency in various trading markets. Table A1[2] lists several examples of these publications and the market indices investigated, in chronological order. Table A2 lists the sample sizes, sample means, and sample standard deviations for daily returns used in those studies, where R_t, daily return at time t, following (Fama, 1965), is defined as

$$R_t = \ln(P_t) - \ln(P_{t-1}) = \ln(P/P_{t-1}) \tag{2}$$

with P_t and P_{t-1} are closing stock or index prices at time t.

In this paper, simulation results from the most popular simple moving average rules — MA(1,5,0) and MA(1,200,0) — are shown. Results from other variations — MA(1,50,0), MA(1,150,0), MA(5,150,0), MA(2,200,0), MA(1,50,1), MA(1,150,1), MA(5,150,1), and MA(2,200,1) — are similar. Our simulation study shows that the *t*-tests applied on Buy and Sell Days returns from previous studies in Tables A1 and A2 result in a false rejection of the efficient market hypothesis.

Various versions of the random walk hypothesis on (logarithmic) returns are discussed and tested in (Campbell et al., 1997). However, other studies on moving average trading rules follow different approaches, i.e. (Brock et al., 1992; Chang et al., 2006; Coutts, Cheung, 2000; Kirkpatrick II, Dahlquist, 2015, and Pring, 2014). These studies attempt to apply the logic of *modus tollens* or *modus tollendo tollens* rather than proving that the time series for returns $R_1, R_2, ..., R_n$ can be versions of random walk. After reclassifying the observations into Buy and Sell groups, say $\{R_{b1}, R_{b2}, ..., R_{bm}\}$ and $\{R_{s1}, R_{s2}, ..., R_{sm}\}$ with $bm + sm = n$, the groups should be two independent samples. If they are, then the *t*-test for independent two-samples can be soundly applied to test whether the two groups have equal means or not. Because $\{R_1, R_2, ..., R_n\}$ and $\{R_{s1}, R_{s2}, ..., R_{sm}\}$ are sub-groups of i.i.d. time series $\{R_1, R_2, ..., R_n\}$, they should have equal means. Therefore, if *t*-tests for independent two samples can show that the Buy and Sell sub-groups have different means, then by the logic of *modus tollens* or *modus tollendo tollens*, the time series for returns $\{R_1, R_2, ..., R_n\}$ will not be i.i.d. In actuality, this is a spurious conclusion.

The reclassification of observations via a moving average trading rule such as the one described in this article or by some other trading rule (such as the trading break-out rule in (Ren et al., 2018)) may not be random. In other words, the sub-groups $\{R_{b1}, R_{b2}, ..., R_{bm}\}$ and $\{R_{s1}, R_{s2}, ..., R_{sm}\}$ for Buy/Sell days may not be independent. Therefore, the *t*-test for independent two samples cannot be applied on $\{R_{b1}, R_{b2}, ..., R_{bm}\}$ and $\{R_{s1}, R_{s2}, ..., R_{sm}\}$. The *t*-tests for independent two-samples are different from the *t*-tests used to test whether or not the time series for returns $R_1, R_2, ..., R_n$ are i.i.d. (Campbell et al., 1997, pp. 33–41; Gabaix et al., 2006). The former erroneously assumes that $R_1, R_2, ..., R_n$ are i.i.d. In this study, we show that the type I error derived from the *t*-tests discussed in the next section are large. Researchers should not use the results of rejections from such *t*-tests to claim that the market is not efficient due to prevalent type I errors.

2. Data analysis

In Fama's seminal 1970 study, market efficiency is classified into "weak", "semi-strong", and "strong" forms. Under the weak form of market efficiency, all historical information is reflected in the stock price today; therefore, technical analysis cannot help investors predict the stock's

[2] All tables marked with 'A' are in the Appendix.

future price behaviors. Under the semi-strong form of market efficiency, all public information is reflected in today's stock price; therefore, investors cannot use the help of technical nor fundamental analysis to gain abnormal returns in the market. Under the strong form of market efficiency, all public and non-public information is reflected in the current stock price; therefore, an investor cannot gain abnormal profits by using any type of information from the market. If the market is weak-form efficient, prices follow a random walk and the daily returns R_t in equation (2) are independently and identically distributed (i.i.d.).

2.1. I.i.d.ness under normal distribution

In the most ideal situation, we assume that for different market indices R_t follow i.i.d. normal probability distributions (Fama, 1965) with the means and standard deviations listed in Table A2. From equation (2) $R_t = \ln\left(P_t / P_{t-1}\right)$, we have $P_t = P_{t-1}e^{R_t}$. Without the loss of generality, we let $P_0 = 100$. Accordingly, from simulated i.i.d. normal daily returns $R_1, R_2, ..., R_n$, we will be able to generate daily price time series $P_1, P_2, ..., P_n$. When we apply the trading rules MA(1,5,0) and MA(1,200,0) to the simulated daily price time series, every trading day from Day 5$^{\text{th}}$ to Day 200$^{\text{th}}$ can be classified or labeled as a Buy or a Sell Day according to equation (1). Results from the *t*-tests are listed in Tables A4 and A5, where

$$T_1 = \frac{\bar{X}_b - \mu_b}{s_b / \sqrt{n}} \text{ for testing } \mathbf{H_0}: \mu = \mu_b \text{ v.s. } \mathbf{H_a}: \mu \neq \mu_b, \quad (3)$$

$$T_2 = \frac{\bar{X}_s - \mu_s}{s_s / \sqrt{n}} \text{ for testing } \mathbf{H_0}: \mu = \mu_s \text{ v.s. } \mathbf{H_a}: \mu \neq \mu_s, \text{ and} \quad (4)$$

$$T_3 = \frac{\bar{X}_b - \bar{X}_s}{\sqrt{s_b^2 / n_b + s_s^2 / n_s}} \text{ for testing } \mathbf{H_0}: \mu_b = \mu_s \text{ v.s. } \mathbf{H_a}: \mu_b \neq \mu_s, \quad (5)$$

μ_b and μ_s are population means, $\bar{X}_b$ and $\bar{X}_s$ are sample means, s_b, s_s are sample standard deviations, and n_b, n_s are sample sizes for daily returns of Buy and Sell Days, respectively.

In our simulation study, we first generate $n-1$ i.i.d. random values for returns, $R_1, R_2, ..., R_{n-1}$, with a mean μ equal to $\bar{X}$, and a standard deviation of σ equal to *s*, where the $\bar{X}$ and *s* are given in the last two columns in Table A2. We can now generate a simulated series of prices P_t, where $P_1 = 100$ and $P_t = P_{t-1}e^{R_t}$.

To illustrate, for the trading rule MA(1,5,0), P_5 is compared to the average of P_1 through P_5. If P_5 is greater or equal to the average of P_1 through P_5, Day 5 is classified or labeled as a Buy Day; otherwise, Day 5 is classified or labeled as a Sell Day. We continue this process from Day 5 to Day *n*. *T*-tests in equations (3)–(5) are then applied to returns in Buy and Sell Days. Partial results from our simulated data for the DJIA can be found in Tables A3.

From Tables A4 and A5, if researchers use the results of *t*-tests to test for market efficiency, then at the 0.01 significance level, we would reject H_0's from the *t*-tests 1% of the time and would accordingly conclude the markets were not efficient; however, the markets are indeed efficient with simulated daily returns from i.i.d. normal probability distributions. Therefore, researchers should not use the results of rejections from *t*-tests to claim that the market is not efficient due to prevalent type I errors.

As shown in Tables A4 and A5, we have obtained similar results of large type I errors from simulated i.i.d. returns for different price indices using various $\mathrm{MA}(S, L, \alpha)$ trading rules.

2.2. I.i.d.ness under *t*-distribution with 3 degrees of freedom

In the last section, the numerical results in Tables A4 and A5 are obtained under the most ideal normality assumptions for the returns as defined in equation (2). However, many works in the literature indicate that the distribution of financial returns typically exhibits deviations from normality and are stylized as heavy-tailed (Embrechts et al., 1997; Gabaix et al., 2006; Gabaix, 2009, and Ibragimov et al., 2015).

The heavy-tailedness for the distributions of financial returns is usually modelled using power laws (PL). Gopikrishnan et al. (1999) established an inverse cubic PL model for stock market returns over 200 million data points. They found that the distribution function of logarithmic returns defined in eq. (2) for the 1000 largest U. S. stocks and several major international indices is $\mathrm{P}\left(|R_t| > x\right) \sim Cx^{-\xi}$ with $C > 0$, $\xi \approx 3$ and when $x \to \infty$ (where $f(x) \sim g(x)$ as $x \to \infty$ means that $\lim_{x\to\infty} f(x)/g(x) = 1$). This relationship holds for positive and negative returns separately. We can repeat our simulation analysis using this new set of assumptions.

For a Student *t*-distribution with three degrees of freedom, *T*(3), it follows the above PL with a tail index $\xi \approx 3$. The expected value of *T*(3) is $\mathrm{E}\left(T(3)\right) = 0$ and its variance is $\mathrm{Var}\left(T(3)\right) = \frac{3}{3-2} = 3$. All k^{th} moments $\mathrm{E}\left(T(3)^k\right)$, where *k* is greater than d.f.=3, do not exist as shown being a variable has a PL exponent $\xi = 3$. We can generate i.i.d. *T*(3) distributions for returns via Microsoft Excel, R, Matlab etc.

Our study shows that if the markets are indeed efficient with simulated daily returns from i.i.d. *T*(3) distributions, researchers should still not use the results of rejections from *t*-tests to claim that the market is not efficient due to prevalent type I errors.

3. Conclusion

Jensen and Bennington (1970) once said that that given enough computer time, we are sure that we can find a mechanical trading rule which “works” on a table of random values — provided of course that we are allowed to test the rule on same table of numbers which we used to discover the rule (p. 470). His brief simulation article shows that almost all of the time the moving average mechanical trading rules work via popularly used *t*-tests. However, the use of *t*-tests for hypotheses in equations (3)–(5) causes huge type I errors by rejecting the claim that markets are efficient when the markets are indeed efficient from simulated i.i.d. normal market price indices. Therefore, researchers should not use the results of rejections from *t*-tests to claim that the market is not efficient due to prevalent type I errors.

We did not address the appropriateness of using the *t*-tests since observations in Buy and Sell Days or between themselves may not be independent. Regards to the independence issue, please refer to (Ren et al., 2018) for a study on the runs test.

References

Armour J., Lofton M., Oyenekan O., Metghalchi M. (2010). Efficient market hypothesis and technical analysis: The Irish stock index. *International Handbook of Academic Research and Teaching*, 10 (1), 71–74.

Bessembinder H., Chan K. (1995). The profitability of technical trading rules in the Asian stock markets. *Pacific-Basin Finance Journal*, 3 (7), 257–284.

Brock W., Lakonishok J., LeBaron B. (1992). Simple technical trading rules and the stochastic properties of stock returns. *Journal of Finance*, 47 (5), 1731–1764.

Campbell J. Y., Lo A. W., MacKinlay A. C. (1997). *The econometrics of financial markets*. *2nd edition*. Princeton: Princeton University Press.

Chang Y., Metghalchi M., Chang C. (2006). Technical trading strategies and cross-national information linkage: The case of Taiwan stock market. *Applied Financial Economics*, 16 (10), 731–743.

Chen C., Metghalchi M. (2012). Weak-form market efficiency: Evidence from the Brazilian stock market. *International Journal of Economics and Finance*, 4 (7), 22–32.

Chen C., Metghalchi M., Garza-Gomez X. (2011). Technical analysis of the Danish stock market. *Business Study Journal*, 3 (2), 107–116.

Coutts J., Cheung K. (2000). Trading rules and tock returns: Some preliminary short run evidence from the Hang Seng, 1985–1997. *Applied Financial Economics*, 10 (6), 578–586.

Embrechts P., Kluppelberg C., Mikosch T. (1997). *Modelling extremal events: For insurance and finance*, vol. 33. New York: Springer Science & Business Media.

Fama E. (1965). The behavior of stock market prices. *Journal of Business*, 38 (1), 34–105.

Fama E. (1970). Efficient capital markets: A review of theory and empirical work. *Journal of Finance*, 25 (2), 383–417.

Gabaix X., Gopikrishnan P., Plerou V, Stanley H. (2006). Institutional investors and stock market volatility. *The Quarterly Journal of Economics*, 121 (2), 461–504.

Gabaix X. (2009). Power laws in economics and finance. *Annual Review of Economics*, 1, 255–293.

Gopikrishnan P., Plerou V., Amaral L., Meyer M., Stanley E. (1999). Scaling of the distribution of fluctuations of financial market indices. *Physical Review E*, 60, 5305–5316.

Hudson R., Dempsey M., Keasey K. (1996). A note on the weak form efficiency of capital markets: The application of simple technical trading rules to UK stock prices 1935–1994. *Journal of Banking and Finance*, 20 (6), 1121–1132.

Ibragimov M., Ibragimov R., Walden J. (2015). Heavy-tailed distributions and robustness in economics and finance. *Lecture Notes in Statistics*, 214. New York: Springer.

Jensen M., Bennington G. (1970). Random walks and technical theories: Some additional evidence. *Journal of Finance*, 25, 469–482.

Kirkpatrick II C., Dahlquist J. (2015). *Technical analysis: The complete resource for financial market technicians. 3rd edition*. New Jersey: FT Press.

Kwon K., Kish R. (2002). Technical trading strategies and return predictability: NYSE. *Applied Financial Economics*, 12 (9), 639–653.

Metghalchi M. (2015). Lost decade, market efficiency and technical trading rules: Evidence from Greece. *The Journal of Prediction Markets*, 9 (1), 15–32.

Metghalchi M., Chang Y. (2003). Profitable technical trading rules for the Italian stock market. *Rivista Internazionale di Scienze Economiche e Commerciali (RISEC)*, 50 (4), 433–450.

Metghalchi M., Garza-Gomez X. (2011). Trading rules for the Abu Dhabi stock index. *Review of Middle East Economics and Finance*, 7 (1), 52–66.

Metghalchi M., Garza-Gomez X. (2013). The use of technical trading rules to predict overall stock price movements: A study on share prices on the Irish stock exchange. *International Journal of Management*, 30 (2), 678–691.

Metghalchi M., Gaza-Gomez X., Chen C., Monsef S. (2005). Market efficiency for S&P 500: 1954–2004. *International Business and Economic Research Journal*, 4 (7), 23–30.

Metghalchi M., Glasure Y., Garza-Gomez X., Chen C. (2007). Profitable technical trading rules for the Austrian stock market. *International Business and Economics Research Journal*, 6 (9), 49–58.

Metghalchi M., Chang Y., Marcucci J. (2008a). Is the Swedish stock market efficient? Evidence from some simple trading rules. *International Review of Financial Analysis*, 17 (3), 475–490.

Metghalchi M., Garza-Gomez X., Glasure Y., Chang Y. (2008b). Are moving average trading rules profitable? Evidence from the Mexican stock market. *The Journal of Applied Business Research*, 24 (1), 115–128.

Metghalchi M., Du J., Ning Y. (2009). Validation of moving average trading rules: Evidence from Hong Kong, Singapore, South Korea, Taiwan. *The Multinational Business Review*, 17 (3), 101–122.

Metghalchi M., Chang Y., Du J. (2011). Technical trading rules for NASDAQ composite index. *International Research Journal of Finance and Economics*, 73, 109–121.

Metghalchi M., Chang Y., Garza-Gomex X. (2012a). Technical analysis of the Taiwan stock market. *International Journal of Economics and Finance*, 4 (1), 90–102.

Metghalchi M., Marcucci J., Chang Y. (2012b). Are moving average trading rules profitable? Evidence from the European stock markets. *Applied Economics*, 44 (12), 1539–1559.

Metghalchi M., Hajilee M., Hayes L. (2013). Market efficiency and profitability of technical trading rules: Evidence from Vietnam. *The Journal of Prediction Markets*, 7 (2), 11–27.

Metghalchi M., Kagochi J., Hayes L. (2014a). Contrarian technical trading rules: Evidence from Nairobi stock index. *The Journal of Applied Business Research*, 30 (3), 833–846.

Metghalchi M., Pinho A., Sarmento A. (2014b). The efficiency of emerging capital markets: The case of Poland. *The Journal of Prediction Markets*, 8 (1), 27–41.

Metghalchi M., Chen C., Hayes L. (2015). History of share prices and market efficiency of the Madrid general stock index. *International Review of Financial Analysis*, 40 (7), 178–184.

Metghalchi M., Chen C., Hajilee M. (2016). Moving average trading rules for NASDAQ composite index. *Applied Finance Letters*, 5 (2), 45–57.

Metghalchi M., Hajilee M., Hayes L. (2018). Return predictability and efficient market hypothesis: Evidence from Iceland. *The Journal of Alternative Investments*, 21 (1), 68–78.

Metghalchi M., Hajilee M., Hayes L. (2019a). Return predictability and market efficiency: Evidence from the Bulgarian stock market. *Eastern European Economics*, 57 (3), 251–268.

Metghalchi M., Hayes L., Niroomand F. (2019b). A technical approach to equity investing in emerging markets. *Review Financial Economics*, 37 (3), 389–403.

Pistole T., Metghalchi M. (2010). Comparison of three technical trading methods vs. buy-and-hold for the S&P 500 market. *2010 SWDSI Conference Proceedings*, PA (153), 1–19.

Pring M. J. (2014). *Technical analysis explained: The successful investor's guide to spotting investment trends and turning points*. *5th edition*. New York: McGraw-Hill.

Ren L., Ren P., Glasure Y. (2018). Is the simple trading range break-out rule profitable from the NASDAQ index? *Benchmarking: An International Journal*, 25 (2), 763–771.

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Appendix

Table A1. Market efficiency studies by using the moving average trading rules

	Market indices	Academic articles
1	DJIA	Brock et al. (1992)
2	Six Asian Stock Market Indices	Bessembinder, Chan (1995)
3	FT30	Hudson et al. (1996)
4	Hang Sheng Market Index	Coutts, Cheung (2000)
5	NYSE	Kwon, Kish (2002)
6	Italian Stock Market Index	Metghalchi, Chang (2003)
7	S&P 500	Metghalchi et al. (2005)
8	Taiwan Stock Market Index	Chang et al. (2006)
9	Austrian Stock Market Index	Metghalchi et al. (2007)
10	Swedish Stock Market Index	Metghalchi et al. (2008a)
11	Mexican Stock Market Index	Metghalchi et al. (2008b)
12	Hong Kong, Singapore, South Korea, Taiwan	Metghalchi et al. (2009)
13	Irish Stock Market Index	Armour et al. (2010)
14	S&P 500	Pistole, Metghalchi (2010)
15	Danish Stock Market Index	Chen et al. (2011)
16	NASDAQ	Metghalchi et al. (2011)
17	Abu Dhabi Stock Index	Metghalchi, Garza-Gomez (2011)
18	Brazilian Stock Market Index	Chen, Metghalchi (2012)
19	Taiwan Stock Market Index	Metghalchi et al. (2012a)
20	Sixteen European Stock Market Indices	Metghalchi et al. (2012b)
21	Irish Stock Market Index	Metghalchi, Garza-Gomez (2013)
22	Vietnam Stock Market Index	Metghalchi et al. (2013)
23	Nairobi Stock Market Index	Metghalchi et al. (2014a)
24	Poland Stock Market Index	Metghalchi et al. (2014b)
25	Greece Stock Market Index	Metghalchi (2015)
26	Madrid Stock Market Index	Metghalchi et al. (2015)
27	NASDAQ	Metghalchi et al. (2016)
28	Bulgarian Market Index	Metghalchi et al. (2019a)
29	Iceland Market Index	Metghalchi et al. (2018)
30	Emerging Market Index	Metghalchi et al. (2019b)

Table A2. Sample size, sample mean, and sample standard deviation studied in various articles

	Market indices	Years	n	$\bar{x}$	s
1	DJIA	1887–1986	25036	0.00017	0.0108
2	Six Asian stock market Indices	1975–1987	≈3950	0.00065	0.0149
3	FT30	1935–1994	14802	0.00026	0.0100
4	Hang Sheng Market Index	1985–1997	3061	0.00074	0.0412
5	NYSE	1962–1996	8685	0.00029	0.0082
6	Italian Stock Market Index	1973–2002	7776	0.00036	0.0136
7	S&P 500	1954–2004	12625	0.00030	0.0092
8	Taiwan Stock Market Index	1983–2002	4895	0.00069	0.0208
9	Austrian Stock Market Index	1990–2006	4312	0.00023	0.0088
10	Swedish Stock Market Index	1986–2004	4878	0.00041	0.0145
11	Mexican Stock Market Index	1988–2004	4175	0.00097	0.0163
12	Hong Kong, Singapore, South Korea, Taiwan	1969–2005	9269	0.00033	0.0199
13	Irish Stock Market Index	1989–2009	4193	0.00013	0.0122
14	S&P 500	1992–2009	4183	0.00017	0.0121
15	Danish Stock Market Index	1993–2010	4455	0.00033	0.0122
16	NASDAQ	1971–2007	9625	0.00034	0.0116
17	Abu Dhabi Stock Market Index	2001–2010	2348	0.00036	0.0229
18	Brazilian Stock Market Index	1996–2001	3838	0.00061	0.0220
19	Taiwan Stock Market Index	1990–2010	5152	0.00014	0.0170
20	Sixteen European Stock Market Indices	1990–2010	5172	0.00020	0.0086
21	Irish Stock Market Index	1988–2010	≈4393	0.00018	0.0117
22	Vietnam Stock Market Index	2002–2012	2732	0.00023	0.0154
23	Nairobi Stock Market Index	2006–2013	1624	0.00000	0.1030
24	Poland Stock Market Index	1998–2013	3815	0.00034	0.0139
25	Greece Stock Market Index	2000–2012	3193	–0.00052	0.0178
26	Madrid Stock Market Index	1995–2012	9913	0.00020	0.0119
27	NASDAQ	1972–2015	11423	0.00033	0.0123
28	Bulgarian Market Index	2003–2018	3724	0.00012	0.0132
29	Iceland Market Index	1999–2016	9166	0.00010	0.0195
30	Emerging Market Index	1988–2017	7436	0.00027	0.0138

Table A3. An illustrated example for simulated DJIA

t	P_t	R_t	MA(5)		Buy	Sell	MA(200)		Buy	Sell
1	100	—								
2	100.26	0.0026								
3	100.05	–0.0021								
4	101.95	0.0188								
5	99.99	–0.0194	100.45	Sell		–0.0194				
6	101.54	0.0155	100.76	Buy	0.0155					
7	102.16	0.0061	101.14	Buy	0.0061					
8	100.87	–0.0127	101.30	Sell		–0.0127				
9	101.72	0.0084	101.26	Buy	0.0084					
10	100.19	–0.0151	101.30	Sell		–0.0151				
⋮										
200	88.87	0.0165	88.61	Buy	0.0165		88.35	Sell		0.0165
201	89.91	0.0116	88.63	Buy	0.0116		88.61	Sell		0.0116
202	90.13	0.0025	89.03	Buy	0.0025		88.86	Sell		0.0025
203	89.35	–0.0087	89.14	Buy	–0.0087		89.00	Sell		–0.0087
204	89.45	0.0011	89.54	Sell		0.0011	89.12	Sell		0.0011
205	89.71	0.0029	89.71	Buy	0.0029		89.29	Sell		0.0029
206	91.04	0.0147	89.94	Buy	0.0147		89.49	Sell		0.0147
207	93.18	0.0232	90.55	Buy	0.0232		89.72	Buy	0.0232	
208	93.58	0.0043	91.39	Buy	0.0043		89.98	Buy	0.0043	
209	94.43	0.0090	92.39	Buy	0.0090		90.23	Buy	0.0090	
210	92.81	–0.0173	93.01	Sell		–0.0173	90.43	Sell		–0.0173
⋮										
25027	1946.52	–0.0122	1961.57	Sell		–0.0122	1825.63	Buy	–0.0122	
25028	1947.01	0.0002	1959.88	Sell		0.0002	1837.44	Buy	0.0002	
25029	1958.50	0.0059	1955.67	Buy	0.0059		1849.50	Buy	0.0059	
25030	2003.94	0.0229	1965.26	Buy	0.0229		1863.13	Buy	0.0229	
25031	2000.58	–0.0017	1971.31	Buy	–0.0017		1876.97	Buy	–0.0017	
25032	2034.44	0.0168	1988.90	Buy	0.0168		1892.45	Buy	0.0168	
25033	2013.60	–0.0103	2002.21	Buy	–0.0103		1907.49	Buy	–0.0103	
25034	2033.39	0.0098	2017.19	Buy	0.0098		1921.66	Buy	0.0098	
25035	2043.25	0.0048	2025.05	Buy	0.0048		1936.96	Buy	0.0048	
25036	2066.45	0.0113	2038.23	Buy	0.0113		1950.98	Buy	0.0113	
n					12627	12405			13512	11325
$\bar{x}$					0.0063	–0.0061			0.0011	–0.0011
s					0.0088	0.0087			0.0107	0.0106
T_1					77.794				10.418	
T_2					–80.818				–12.378	
T_3					92.055				16.163	

Table A4. Results for *t*-tests from MA(1,5,0) to 30 simulated closing or stock index prices where their returns following i.i.d. normal distributions

	n_b	n_s	$\bar{x}_b$	$\bar{x}_s$	s_b	s_s	T_1	T_2	T_3
1	12627	12405	0.0063	–0.0061	0.0088	0.0087	77.79 (0.0000)	–80.82 (0.0000)	92.05 (0.0000)
2	2087	1860	0.0089	–0.0084	0.0120	0.0118	31.59 (0.0000)	–33.20 (0.0000)	37.40 (0.0000)
3	7789	7009	0.0059	–0.0057	0.0082	0.0080	60.63 (0.0000)	–62.36 (0.0000)	71.29 (0.0000)
4	1536	1521	0.0243	–0.0230	0.0334	0.0328	27.64 (0.0000)	–28.22 (0.0000)	32.34 (0.0000)
5	4368	4313	0.0048	–0.0048	0.0067	0.0067	44.46 (0.0000)	–50.26 (0.0000)	54.68 (0.0000)
6	4096	3676	0.0080	–0.0076	0.0110	0.0109	44.29 (0.0000)	–44.37 (0.0000)	51.10 (0.0000)
7	6501	6120	0.0054	–0.0053	0.0076	0.0074	54.24 (0.0000)	–58.82 (0.0000)	65.67 (0.0000)
8	2546	2345	0.0119	–0.0122	0.0172	0.0166	32.80 (0.0000)	–37.55 (0.0000)	40.76 (0.0000)
9	2171	2137	0.0054	–0.0051	0.0072	0.0071	33.28 (0.0000)	–34.89 (0.0000)	39.07 (0.0000)
10	2482	2362	0.0084	–0.0085	0.0119	0.0120	33.62 (0.0000)	–36.23 (0.0000)	40.37 (0.0000)
11	2278	1893	0.0097	–0.0094	0.0134	0.0128	30.82 (0.0000)	–35.17 (0.0000)	38.48 (0.0000)
12	4731	4534	0.0117	–0.0115	0.0161	0.0160	48.56 (0.0000)	–49.77 (0.0000)	56.62 (0.0000)
13	2164	2025	0.0073	–0.0070	0.0100	0.0101	33.36 (0.0000)	–31.64 (0.0000)	38.00 (0.0000)
14	2193	1986	0.0070	–0.0070	0.0098	0.0099	32.50 (0.0000)	–32.36 (0.0000)	37.45 (0.0000)
15	2259	2192	0.0073	–0.0069	0.0101	0.0097	32.68 (0.0000)	–35.02 (0.0000)	39.14 (0.0000)
16	5018	4603	0.0068	–0.0067	0.0095	0.0096	48.68 (0.0000)	–49.54 (0.0000)	56.76 (0.0000)
17	1177	1167	0.0141	–0.0130	0.0189	0.0183	24.94 (0.0000)	–25.01 (0.0000)	28.53 (0.0000)
18	2048	1786	0.0118	–0.0111	0.0159	0.0156	31.81 (0.0000)	–31.82 (0.0000)	36.46 (0.0000)
19	2506	2642	0.0100	–0.0098	0.0137	0.0138	35.95 (0.0000)	–36.99 (0.0000)	42.12 (0.0000)
20	2468	2700	0.0514	–0.0503	0.0680	0.0690	37.36 (0.0000)	–38.04 (0.0000)	42.81 (0.0000)
21	2250	2139	0.0134	–0.0134	0.0184	0.0188	34.16 (0.0000)	–33.37 (0.0000)	38.70 (0.0000)
22	1449	1279	0.0088	–0.0089	0.0124	0.0126	26.23 (0.0000)	–25.76 (0.0000)	29.48 (0.0000)

End of the table A4

	n_b	n_s	$\bar{x}_b$	$\bar{x}_s$	s_b	s_s	T_1	T_2	T_3
23	775	845	0.0599	–0.0591	0.0845	0.0824	19.76 (0.0000)	–20.86 (0.0000)	23.62 (0.0000)
24	1992	1819	0.0082	–0.0080	0.0114	0.0113	30.93 (0.0000)	–31.58 (0.0000)	36.05 (0.0000)
25	1511	1678	0.0102	–0.0102	0.0146	0.0142	28.47 (0.0000)	–27.74 (0.0000)	32.48 (0.0000)
26	4993	4916	0.0070	–0.0070	0.0097	0.0096	49.58 (0.0000)	–52.09 (0.0000)	58.43 (0.0000)
27	5845	5574	0.0072	–0.0070	0.0101	0.0101	51.89 (0.0000)	–54.32 (0.0000)	61.94 (0.0000)
28	1917	1803	0.0077	–0.0077	0.0106	0.0109	31.20 (0.0000)	–30.60 (0.0000)	35.74 (0.0000)
29	4605	4557	0.0113	–0.0115	0.0156	0.0160	48.63 (0.0000)	–48.93 (0.0000)	56.26 (0.0000)
30	3858	3574	0.0082	–0.0081	0.0113	0.0111	43.25 (0.0000)	–45.12 (0.0000)	50.94 (0.0000)

Note. P-values are in the parenthesis, and all are significant at $\alpha = 0.01$.

Table A5. Results for *t*-tests from MA(1,200,0) to 30 simulated closing or stock index prices where their returns following i.i.d. normal distributions

	n_b	n_s	$\bar{x}_b$	$\bar{x}_s$	s_b	s_s	T_1	T_2	T_3
1	13512	11325	0.0011	–0.0011	0.0107	0.0106	10.42 (0.0000)	–12.38 (0.0000)	16.16 (0.0000)
2	2849	903	0.0015	–0.0015	0.0147	0.0145	2.93 (0.0034)	–4.48 (0.0000)	5.34 (0.0000)
3	10726	3877	0.0010	–0.0011	0.0100	0.0098	7.27 (0.0000)	–8.69 (0.0000)	11.21 (0.0000)
4	1755	1107	0.0037	–0.0035	0.0399	0.0410	3.15 (0.0016)	–3.44 (0.0000)	4.65 (0.0000)
5	4392	4094	0.0007	–0.0008	0.0082	0.0082	3.36 (0.0008)	–8.14 (0.0000)	8.18 (0.0000)
6	5127	2450	0.0015	–0.0013	0.0133	0.0136	5.99 (0.0000)	–5.94 (0.0000)	8.28 (0.0000)
7	7483	4943	0.0009	–0.0008	0.0093	0.0090	5.70 (0.0000)	–8.69 (0.0000)	10.32 (0.0000)
8	2650	2046	0.0020	–0.0020	0.0209	0.0203	3.32 (0.0009)	–5.95 (0.0000)	6.64 (0.0000)
9	2334	1779	0.0010	–0.0008	0.0090	0.0086	4.12 (0.0000)	–5.04 (0.0000)	6.50 (0.0000)
10	2609	2040	0.0012	–0.0012	0.0145	0.0147	2.92 (0.0035)	–5.01 (0.0000)	5.70 (0.0000)
11	3091	885	0.0019	–0.0018	0.0162	0.0156	3.21 (0.0012)	–5.34 (0.0000)	6.23 (0.0000)

End of the table A5

	n_b	n_s	$\bar{x}_b$	$\bar{x}_s$	s_b	s_s	T_1	T_2	T_3
12	4929	4141	0.0020	–0.0016	0.0198	0.0196	6.07 (0.0000)	–6.47 (0.0000)	8.87 (0.0000)
13	2631	1363	0.0014	–0.0014	0.0125	0.0118	5.22 (0.0000)	–4.67 (0.0000)	6.88 (0.0000)
14	2403	1581	0.0013	–0.0011	0.0120	0.0122	4.45 (0.0000)	–4.22 (0.0000)	6.08 (0.0000)
15	2457	1799	0.0012	–0.0011	0.0123	0.0119	3.44 (0.0006)	–4.96 (0.0000)	6.00 (0.0000)
16	6349	3077	0.0011	–0.0012	0.0116	0.0118	5.56 (0.0000)	–7.44 (0.0000)	9.28 (0.0000)
17	1466	683	0.0022	–0.0025	0.0230	0.0231	3.07 (0.0022)	–3.19 (0.0015)	4.36 (0.0000)
18	2739	900	0.0021	–0.0016	0.0195	0.0194	3.99 (0.0007)	–3.41 (0.0001)	4.95 (0.0000)
19	2129	2824	0.0020	–0.0019	0.0170	0.0168	5.05 (0.0000)	–6.44 (0.0000)	8.03 (0.0000)
20	1714	3259	0.0093	–0.0069	0.0861	0.0844	4.36 (0.0000)	–4.81 (0.0000)	6.34 (0.0000)
21	2438	1756	0.0020	–0.0021	0.0226	0.0233	4.05 (0.0001)	–4.17 (0.0000)	5.80 (0.0000)
22	1815	718	0.0016	–0.0022	0.0149	0.0163	4.03 (0.0001)	–4.05 (0.0001)	5.52 (0.0000)
23	559	866	0.0107	–0.0096	0.1017	0.1013	2.50 (0.0128)	–2.78 (0.0055)	3.69 (0.0000)
24	2288	1328	0.0016	–0.0014	0.0138	0.0141	4.37 (0.0000)	–4.54 (0.0000)	6.25 (0.0000)
25	1002	1992	0.0015	–0.0016	0.0180	0.0173	3.47 (0.0005)	–2.80 (0.0052)	4.44 (0.0000)
26	5440	4274	0.0010	–0.0011	0.0118	0.0120	5.14 (0.0000)	–7.17 (0.0000)	8.78 (0.0000)
27	6444	4780	0.0013	–0.0012	0.0124	0.0120	6.50 (0.0000)	–8.85 (0.0000)	10.93 (0.0000)
28	2137	1388	0.0011	–0.0013	0.0133	0.0131	3.54 (0.0004)	–4.02 (0.0001)	5.36 (0.0000)
29	4271	4696	0.0020	–0.0020	0.0191	0.0197	6.60 (0.0000)	–7.26 (0.0000)	9.80 (0.0000)
30	4343	2894	0.0014	–0.0013	0.0139	0.0136	5.45 (0.0000)	–6.36 (0.0000)	8.38 (0.0000)

Note. P-values are in the parenthesis, and all are significant at $\alpha = 0.01$.

Table A6. Results for *t*-tests from MA(1,5,0) to 30 simulated closing or stock index prices where their returns following i.i.d. fat-tailed *T*(3) distributions

	n_b	n_s	$\bar{x}_b$	$\bar{x}_s$	s_b	s_s	T_1	T_2	T_3
1	12956	12076	0.0091	–0.0088	0.0191	0.0187	51.39 (0.0000)	–51.89 (0.0000)	74.87 (0.0000)
2	2104	1842	0.0121	–0.0122	0.0216	0.0408	24.14 (0.0000)	–13.59 (0.0000)	22.90 (0.0000)
3	7724	7074	0.0086	–0.0086	0.0195	0.0233	37.16 (0.0000)	–32.41 (0.0000)	48.52 (0.0000)
4	1567	1490	0.0342	–0.0310	0.0616	0.0608	20.42 (0.0000)	–21.21 (0.0000)	29.44 (0.0000)
5	4556	4125	0.0065	–0.0066	0.0118	0.0121	35.55 (0.0000)	–36.69 (0.0000)	51.08 (0.0000)
6	4165	3607	0.0111	–0.0114	0.0188	0.0202	35.71 (0.0000)	–35.67 (0.0000)	50.34 (0.0000)
7	6699	5922	0.0076	–0.0075	0.0174	0.0148	33.28 (0.0000)	–41.64 (0.0000)	52.60 (0.0000)
8	2522	2369	0.0168	–0.0172	0.0290	0.0511	28.53 (0.0000)	–16.70 (0.0000)	28.38 (0.0000)
9	2215	2093	0.0073	–0.0072	0.0132	0.0135	25.06 (0.0000)	–25.22 (0.0000)	35.55 (0.0000)
10	2492	2382	0.0117	–0.0116	0.0193	0.0211	29.52 (0.0000)	–27.53 (0.0000)	40.1963 (0.0000)
11	2180	1991	0.0135	–0.0132	0.0230	0.0238	25.79 (0.0000)	–26.26 (0.0000)	36.79 (0.0000)
12	4643	4622	0.0167	–0.0166	0.0333	0.0415	33.95 (0.0000)	–27.33 (0.0000)	42.56 (0.0000)
13	2107	2082	0.0099	–0.0101	0.0158	0.0194	28.88 (0.0000)	–23.60 (0.0000)	36.52 (0.0000)
14	2073	2106	0.0098	–0.0102	0.0178	0.0204	25.78 (0.0000)	–22.37 (0.0000)	33.83 (0.0000)
15	2361	2090	0.0099	–0.0094	0.0188	0.0161	23.40 (0.0000)	–28.96 (0.0000)	36.80 (0.0000)
16	4937	4684	0.0093	–0.0093	0.0171	0.0169	37.18 (0.0000)	–38.63 (0.0000)	53.61 (0.0000)
17	1176	1168	0.0189	–0.0200	0.0351	0.0399	18.9789 (0.0000)	–16.73 (0.0000)	25.09 (0.0000)
18	1891	1943	0.0174	–0.0178	0.0307	0.0335	25.25 (0.0000)	–22.90 (0.0000)	33.97 (0.0000)
19	2639	2509	0.0151	–0.0143	0.0532	0.0424	13.82 (0.0000)	–17.81 (0.0000)	21.97 (0.0000)
20	2626	2542	0.0068	–0.0068	0.0117	0.0127	29.27 (0.0000)	–27.60 (0.0000)	40.11 (0.0000)
21	2165	2224	0.0093	–0.0097	0.0169	0.0189	26.53 (0.0000)	–23.47 (0.0000)	35.21 (0.0000)
22	1377	1351	0.0123	–0.0132	0.0202	0.0478	23.14 (0.0000)	–9.87 (0.0000)	18.05 (0.0000)

End of the table A6

	n_b	n_s	$\bar{x}_b$	$\bar{x}_s$	s_b	s_s	T_1	T_2	T_3
23	833	787	0.0820	–0.0805	0.1353	0.1352	16.81 (0.0000)	–17.35 (0.0000)	24.16 (0.0000)
24	1999	1812	0.0108	–0.0111	0.0187	0.0200	25.02 (0.0000)	–24.48 (0.0000)	34.91 (0.0000)
25	1517	1672	0.0140	–0.0146	0.0249	0.0263	23.44 (0.0000)	–21.14 (0.0000)	31.52 (0.0000)
26	5846	5573	0.0099	–0.0099	0.0178	0.0247	41.74 (0.0000)	–30.75 (0.0000)	49.16 (0.0000)
27	5752	5667	0.0097	–0.0104	0.0175	0.0298	43.05 (0.0000)	–25.53 (0.0000)	43.76 (0.0000)
28	1969	1842	0.0102	–0.0106	0.0178	0.0190	25.19 (0.0000)	–24.42 (0.0000)	35.01 (0.0000)
29	4623	4539	0.0160	–0.0160	0.0291	0.0302	37.06 (0.0000)	–36.10 (0.0000)	51.70 (0.0000)
30	3785	3647	0.0113	–0.0113	0.0190	0.0202	35.81 (0.0000)	–34.33 (0.0000)	49.51 (0.0000)

Note. P-values are in the parenthesis, and all are significant at $\alpha = 0.01$.

Table A7. Results for *t*-tests from MA(1,200,0) to 30 simulated closing or stock index prices where their returns following i.i.d. fat-tailed *T*(3) distributions

	n_b	n_s	$\bar{x}_b$	$\bar{x}_s$	s_b	s_s	T_1	T_2	T_3
1	15346	9491	0.0019	–0.0019	0.0201	0.0219	11.55 (0.0000)	–8.39 (0.0000)	13.55 (0.0000)
2	2565	1186	0.0024	–0.0030	0.0247	0.0497	3.47 (0.0003)	–2.61 (0.0046)	3.58 (0.0003)
3	9299	5304	0.0019	–0.0020	0.0203	0.0268	6.95 (0.0000)	–6.67 (0.0000)	9.24 (0.0000)
4	1912	950	0.0068	–0.0066	0.0627	0.0803	3.09 (0.0010)	–3.45 (0.0000)	4.51 (0.0000)
5	5230	3256	0.0013	–0.0011	0.0137	0.0133	4.91 (0.0000)	–6.19 (0.0000)	7.90 (0.0000)
6	4710	2867	0.0024	–0.0021	0.0219	0.0232	5.51 (0.0000)	–6.46 (0.0000)	8.47 (0.0000)
7	7834	4592	0.0015	–0.0013	0.0183	0.0169	4.87 (0.0000)	–7.28 (0.0000)	8.71 (0.0000)
8	2642	2054	0.0031	–0.0034	0.0330	0.0567	4.26 (0.0000)	–3.01 (0.0013)	4.62 (0.0000)
9	2365	1748	0.0016	–0.0015	0.0154	0.0148	4.15 (0.0000)	–4.81 (0.0000)	6.35 (0.0000)
10	2740	1939	0.0021	–0.0021	0.0225	0.0247	4.04 (0.0000)	–4.27 (0.0000)	5.85 (0.0000)
11	2392	1584	0.0025	–0.0023	0.0260	0.0278	3.35 (0.0004)	–4.32 (0.0000)	5.46 (0.0000)

End of the table A7

	n_b	n_s	$\bar{x}_b$	$\bar{x}_s$	s_b	s_s	T_1	T_2	T_3
12	4993	4077	0.0036	–0.0039	0.0372	0.0451	6.55 (0.0000)	–5.70 (0.0000)	8.48 (0.0000)
13	2107	2082	0.0099	–0.0101	0.0158	0.0194	28.88 (0.0000)	–23.60 (0.0000)	36.52 (0.0000)
14	1843	2141	0.0019	–0.0020	0.0225	0.0207	4.20 (0.0000)	–3.88 (0.0000)	5.71 (0.0000)
15	2946	1310	0.0021	–0.0021	0.0203	0.0190	3.43 (0.0003)	–5.56 (0.0000)	6.52 (0.0000)
16	4878	4548	0.0022	–0.0018	0.0199	0.0187	6.87 (0.0000)	–7.39 (0.0000)	10.08 (0.0000)
17	829	1320	0.0038	–0.0030	0.0414	0.0434	3.01 (0.0013)	–2.14 **(0.0161)**	3.68 (0.0002)
18	1517	2122	0.0035	–0.0033	0.0332	0.0390	4.60 (0.0000)	–3.43 (0.0003)	5.68 (0.0000)
19	3114	1839	0.0036	–0.0026	0.0517	0.0284	3.01 (0.0013)	–5.16 (0.0000)	5.44 (0.0000)
20	2544	2429	0.0014	–0.0012	0.0138	0.0143	4.51 (0.0000)	–4.40 (0.0000)	6.29 (0.0000)
21	1698	2496	0.0018	–0.0020	0.0195	0.0206	4.38 (0.0000)	–4.14 (0.0000)	6.02 (0.0000)
22	1011	1522	0.0026	–0.0024	0.0251	0.0469	3.75 (0.0000)	–1.74 **(0.0410)**	3.51 (0.0004)
23	818	607	0.0211	–0.0219	0.1577	0.1641	3.27 (0.0005)	–3.76 (0.0001)	4.98 (0.0000)
24	1936	1680	0.0020	–0.0019	0.0222	0.0220	3.158 (0.0008)	–4.33 (0.0000)	5.32 (0.0000)
25	1053	1941	0.0031	–0.0031	0.0291	0.0295	4.53 (0.0000)	–3.25 (0.0006)	5.58 (0.0000)
26	6423	4801	0.0021	–0.0021	0.0201	0.0272	7.28 (0.0000)	–5.87 (0.0000)	8.86 (0.0000)
27	5133	6091	0.0019	–0.0020	0.0201	0.0308	7.63 (0.0000)	–4.41 (0.0000)	8.01 (0.0000)
28	1831	1785	0.0020	–0.0022	0.0206	0.0213	3.84 (0.0001)	–4.65 (0.0000)	6.01 (0.0000)
29	4324	4643	0.0035	–0.0030	0.0331	0.0342	6.55 (0.0000)	–6.37 (0.0000)	9.13 (0.0000)
30	3932	3305	0.0021	–0.0022	0.0221	0.0231	5.51 (0.0000)	–5.93 (0.0000)	8.09 (0.0000)

Note. All are significant at $\alpha = 0.01$ except for test statistic T_2 in rows 17 and 22 as bolded.