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Forecasting the Value-at-Risk of energy commodities: A comparison of models and alternative distribution functions

Economic agents need to adequately control, and measure potential financial losses associated with commodity price swings in the futures market. One of the ways to anticipate possible price swings is to measure Value-at-Risk (VaR). In its parametric form, the VaR calculation uses the volatility of a financial asset as a parameter to measure risk. Volatility is the essence of VaR calculation and should be estimated as accurately as possible. The importance of precision in volatility estimation has made heteroskedastic models and their forms of application has evolved significantly in recent years. In this context, this study aimed to verify if the incorporation of several additional parameters in the mathematical expression of the models and the use of different density functions improves the predictive capacity of the conditional variance when used in the measurement of the VaR of the energy commodities in the futures market. The results showed that the use of mathematically more complex structures is not related to better predictions of VaR. However, the use of different density functions allowed the models to fit more adequately to the data, leading to more realistic predictions of conditional variance.

Keywords: Value-at-Risk; heteroscedasticity; GARCH; distribution functions; forecast.
JEL classification: C46; C53; G13; G32; P18.

1. Introduction

Commodity futures markets represent a significant share of global financial markets, and recent fluctuations in energy commodity prices, as they play an important role in the modern economy, have generated a great deal of concern among various economic agents, such as public policy makers, investors, producers, consumers, financial institutions, and companies operating in the energy markets (Billio et al., 2018; Wang, Li, 2018). Explanations for these unusual fluctuations in energy commodity prices can be diverse. Laporta et al. (2018) point out that business cycles, public policies and the speculative behavior of some market participants can cause short-term imbalances in the supply and demand of commodities, leading to sharp price swings.

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Gómez-Valle et al. (2017) mention that the significant oscillations in commodity prices in the last decade have partly resulted from the fact that some financial companies have adopted diversification strategies of the investment portfolio using commodities as protection. This is quite coherent since risk management is indispensable for the survival of companies and portfolio managers. Knowing how to manage risk is a must for almost all countries and sectors of economic activity, as their knowledge allows companies to be more competitive in the market and not be thrown out by their global competitors. In this way, Billio et al. (2018) show that the high volatility of commodity prices has led to numerous studies in the literature on various ways of hedging to hedge against large variations in energy prices, especially those derived from oil. Hedge refers to the positioning with respect to a given financial asset to mitigate the adverse effect resulting from price changes of another complementary asset (see, e.g., (Ahmad et al., 2018; Ardia et al., 2019; Batten et al., 2017; Caporale, Zekokh, 2019; Junttila et al., 2018; Maciel, 2021; Shrestha et al., 2018; Siu, 2021; Wang et al., 2021; Wu et al., 2020; Zheng, Osmer, 2018)). In this context, energy commodity futures contracts have attracted the attention of several economic agents since they can serve both for speculative purposes and for protection purposes in the real economy.

For these and other reasons, it is fundamental to model the behavior of commodity prices and to find tools capable of quantifying the risks inherent in the market. One of the ways used to anticipate possible sharp fluctuations in the prices of financial assets is the measurement of Value-at-Risk (VaR). VaR is widely used in the literature for this purpose (see for example (Berger, Gençay, 2018; Liu et al., 2018; Sampid et al., 2018; Zhang et al., 2018a, b)). In its parametric form, the VaR calculation uses the volatility of a financial asset or asset portfolio as a parameter to measure risk. It can be said that volatility is the essence of the VaR calculation in its parametric approach and should be estimated as accurately as possible. Di Sanzo (2018) emphasizes that it is quite common in the literature to use models from the Autogressive Conditional Heteroscedasticity (ARCH) family for this purpose, since the time series may present a conditional variance that is not constant over time. For this reason, with each passing year, new models, which consider conditional variance a stochastic process conditioned by its own past values, have been developed and validated, both from a theoretical perspective and from an empirical perspective. These new models have more complex mathematical structures, which seek to capture stylized facts such as nonlinearity, asymmetry, and long memory properties, to improve the estimation of volatility. In fact, these newer, more sophisticated models are expected to outperform mathematically simplified models.

Although there is a great deal of scholarly research on ARCH family models, few focus on the importance of probability density functions in estimating volatilities (with the notable exception of (Gunay, Khaki, 2018; Hasanov et al., 2018; Lyu et al., 2017; Slim et al., 2017)). This oversight may have led to an overestimation or underestimation of the volatility used to measure the VaR, leading to a poor specification of the risk of the financial asset. For example, energy commodity series may present statistical probability distributions with different forms of a Gaussian distribution and, therefore, it is necessary to take into account parameters that refer to the position of the curve in relation to the origin, to the dispersion of the curve, the degree of asymmetry, kurtosis, the degree of elevation or flattening of the distribution, and other factors. There are some probability density functions that take all these parameters into account in their mathematical structure and may be more appropriate to describe the relative probability of a random variable assuming a given value. Therefore, once this importance is exposed, it is expected that the use of different probability distributions in the estimation of heteroscedastic models will improve the estimation of volatility used in the calculation of the VaR of energy commodities.

Considering the previous exposed factors, the objective of this study is to verify if the incorporation of several additional parameters in the mathematical expression of the ARCH models and in the density functions used by them improves the predictive capacity of the conditional variance when used in the VaR measurement of energy commodities in the futures market. This work innovates and differs from the others for several reasons, namely:

(a) investigates the predictive ability of heterogeneous ARCH family models for conditional variance, through VaR predictions in a moving window size;

(b) analyzes the relevance of using a wide range of distribution functions in estimating volatility for the VaR calculation;

(c) uses series of energy commodities little mentioned in scientific works.

The academic literature on modeling and forecasting volatility in the commodities market is limited and has focused mainly on non-renewable energy and precious metals, see for example (Billio et al., 2018; Chang et al., 2018; Di Sanzo, 2018; Gómez-Valle et al., 2017; Guan, An, 2017; Gunay, Khaki, 2018; Laporta et al., 2018; Lyócsa, Molnár, 2018; Miao et al., 2017; Muteba Mwamba, Mwambi, 2021; Nademi, Nademi, 2018; Niu, Wang, 2017; Sanders, Irwin, 2014; Wang, Li, 2018; Youssef et al., 2015). A few studies have contributed to the literature on studies of biofuel feedstocks, although this issue is relevant (with the notable exception of (Hasanov et al., 2016, 2018)). Currently, biofuels are drawing attention as promising and sustainable sources to replace conventional fuels. Legislative restrictions on the use of fossil fuels, aimed at reducing greenhouse gases, are helping in this process (Çelebi, Aydın, 2019). Thus, to fill this gap, this paper proposes to study the Value-at-Risk of commodities that serve as feedstock for biofuel production. Soybean and maize oil were chosen because they present a higher volume of production worldwide and because they are the most used in the production of biofuels (Hasanov et al., 2018). For the purpose of comparison across different markets, this study also performs VaR forecasts of two non-renewable and commonly studied energy commodities, namely: Crude Oil and Natural Gas.

In addition to this first section containing the introduction, the remainder of this study is organized as follows: section 2 presents the concept and method of calculating the VaR measure, which will be used to measure the market risk of energy commodities, provides the characteristics and the mathematical formulas of the heteroscedastic models, and exposes the different distributions of statistical probability used; section 3 sets out the database and proposed method to meet the previously established objective; section 4 explores the empirical evidence found and makes some comparisons with similar published works; and, finally, section 5 sets out the final conclusions of the paper.

2. Empirical models

2.1. Value-at-Risk

The VaR specification, valid for any discrete or continuous probability distribution function, can be expressed by probabilistic terms. Once that r_t represents the return series of a financial asset in time *t*, the $\mathrm{VaR}_t(\alpha)$ in the percentile $(1-\alpha)$ is defined by $\Pr\left(r_t \le \mathrm{VaR}_t(\alpha)\right) = \alpha$, which computes the return probability in time *t* being lower than, or equal to $\mathrm{VaR}_t(\alpha)$, given a significance level $\alpha \in (0,1)$. In probabilistic terms, the VaR is simply a quantile of the distribution of loss (McNeil et al., 2005, p. 38).

The VaR calculation can be considerably simplified if a parametric approach is used. To calculate the VaR, through the parametric approach, it is necessary to transform r_t into a standard random variable ε_t, which has zero mean and unit standard deviation. While it is supposed that r_t follows a process $r_t = \mu + \sigma_t \varepsilon_t$, where ε_t has the cumulative distribution function Φ with zero mean and unit variance, the VaR measure might be computed through the following mathematical expression: $\mathrm{VaR}_t(\alpha) = \mu + \sigma_t \Phi^{-1}(\alpha)$, where $\Phi^{-1}(\alpha)$ represents the α-quantile of the distribution function Φ, μ expresses the mean and σ_t symbolizes the standard deviation (volatility) of the time series that can be estimated through a statistical model.

2.2. Heteroscedastic models

Briefly described in this topic, this study uses five heterogeneous models of conditional variance. It is believed that the models used have the capacity to estimate the volatility of commodities adequately to measure the VaR. The first and simplest model to be described refers to the *Generalized ARCH* (GARCH) model developed by Bollerslev (1986). This model expresses conditional variance as a linear function of the square of the past errors and the past values of the time series itself. Let υ_t be a sequence of independent and identically distributed (i.i.d.) random variables, with a probability distribution function η, a process ε_t is called GARCH(p, q) if $\varepsilon_t = \sigma_t \upsilon_t$, where σ_t^2 represents the conditional variance and is a function of $\{\varepsilon_s, s < t\}$ which can be described by the following mathematical equation:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \quad t \in \mathbb{Z}, \tag{1}$$

where $\varepsilon_{t-i} = r_{t-i} - \mu$, for $i = 1, 2, \ldots, p$ in time $t = 1, 2, \ldots, T$, $\mu \in \mathbb{R}$ and expresses the mean of r_t, α_0 is a constant higher than zero, $0 \le \alpha_i < 1$, $\forall i = 1, 2, \ldots, p$ and $0 \le \beta_j < 1$, $\forall j = 1, 2, \ldots, q$ with $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$ to preserve stationarity and positivity in the conditional variance. Equation (1) may be described in a more compact way as: $\sigma_t^2 = \alpha_0 + \alpha(B)\varepsilon_t^2 + \beta(B)\sigma_t^2$, where $t \in \mathbb{Z}$, B the offset operator ($B^i \varepsilon_t^2 = \varepsilon_{t-i}^2$ and $B^i \sigma_t^2 = \sigma_{t-i}^2$, $\forall i \in \mathbb{Z}$), $\alpha(\cdot)$ and $\beta(\cdot)$ are the polynomials of degree p and q, represented, respectively, by $\alpha(B) = \sum_{i=1}^{p} \alpha_i B^i$ and $\beta(B) = \sum_{j=1}^{q} \beta_j B^j$. It is understandable that in the case where $\beta(\varepsilon) \equiv 0$, the time series $\{r_t\}$ is summarized into an ARCH process, defined by $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2$. It is clearly seen that the deficiency of this model refers to the inability to quantify asymmetric impacts on volatility. Trying to solve this deficiency Nelson (1991) proposed the *Exponential GARCH* (EGARCH) model. Being υ_t sequence of i.i.d. random variables, with $\mathrm{E}(\upsilon_t) = 0$ and $\mathrm{Var}(\upsilon_t) = 1$, a process ε_t is denominated of EGARCH(p, q) if $\varepsilon_t = \sigma_t \upsilon_t$, where σ_t^2 represents the conditional variance and is a function of $\{\varepsilon_s, s < t\}$, that might be described by the mathematical equation (2):

$$\ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \left[\alpha_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} + \gamma_i \left(\left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| - \mathrm{E} \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| \right) \right] + \sum_{j=1}^{q} \beta_j \ln(\sigma_{t-j}^2), \tag{2}$$

where $\alpha_0, \alpha_i, \gamma_i$ and $\beta_j \in \mathbb{R}$ and the parameter γ_i represents the shocks asymetric response. In the case $\gamma_i < 0$, that is, if $\varepsilon_{t-1} < 0$, the impact of negative shocks over the future volatility will be higher than that of negative shocks of the same magnitude. Unlike the classical GARCH model, the parameters α_i, β_i and γ_i have no constraints imposed for positivity in the conditional variance, since the model uses a logarithmic specification to be estimated, preventing the variance from being negative.

An alternative way to model the impact of asymmetric shocks was proposed by Glosten et al. (1993) through the model called GJR-GARCH. Let υ_t be a sequence of i.i.d. random variables with $\mathrm{E}(\upsilon_t) = 0$ and $\mathrm{Var}(\upsilon_t) = 1$, a process ε_t is called of GJR-GARCH (*p*, *q*) if $\varepsilon_t = \sigma_t \upsilon_t$, where σ_t^2 represents the conditional variance and is a function of $\{\varepsilon_s, s < t\}$, described by equation:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \left[\alpha_i \varepsilon_{t-i}^2 + \gamma_i \varepsilon_{t-i}^2 \mathrm{I}(\varepsilon_{t-i} < 0) \right] + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \quad (3)$$

where $\mathrm{I}(\cdot)$ represents a function which assumes a unit value if $\varepsilon_{t-i} < 0$, and value zero if $\varepsilon_{t-i} > 0$, $\forall i = 1, 2, \ldots, p$. Notice that due to the presence of $\mathrm{I}(\cdot)$, the model persistency specification depends over the asymmetry of the conditional distribution used to model ε_t: $P = \sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j + \sum_{i=1}^{p} \gamma_i \mathrm{P}(\varepsilon_{t-i} \le 0)$, where $\mathrm{P}(\varepsilon_{t-i} \le 0)$ represents the probability to observe negative shocks. Besides, the conditions $\alpha_0 > 0$, $\alpha_i \ge 0$, $\gamma_i \ge 0$, $\forall i = 1, 2, \ldots, p$ and $\beta_j \ge 0$, $\forall j = 1, 2, \ldots, q$ are needed to ensure that the conditional variance is positive.

Another model used in the present study is the *Asymmetric Power ARCH* (APARCH), proposed by Ding et al. (1993). This model presents a flexible formula, capable of producing, with some restrictions, the main characteristics of some of the models mentioned previously. Being υ_t a sequence of i.i.d. random variables with $\mathrm{E}(\upsilon_t) = 0$ and $\mathrm{Var}(\upsilon_t) = 1$, a process ε_t is denominated of APARCH (*p*, *q*) if $\varepsilon_t = \sigma_t \upsilon_t$, where σ_t^2 represents the conditional variance and is a function of $\{\varepsilon_s, s < t\}$, that can be described by equation:

$$\sigma_t^\delta = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i} \right)^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta, \quad (4)$$

where $\delta \in \mathbb{R}^+$ and increases the model flexibility, allowing the selection of an arbitrary power. Additionally, $\alpha_0 > 0$, $\alpha_i \ge 0$, $\forall i = 1, 2, \ldots, p$, $\gamma_i \ge 0$, $\forall i = 1, 2, \ldots, p$ and $\beta_j \ge 0$, $\forall j = 1, 2, \ldots, q$ are the imposed restrictions in the parameters to ensure the positivity of the conditional variance.

Finally, the last model used is the *Component GARCH* (CGARCH), developed by Engle, Lee (1993). This model decomposes the conditional variance in two components: one permanent component and another transitory, in order to investigate the long and short run components of the financial volatility series. Being υ_t a sequence of i.i.d. random variables with $\mathrm{E}(\upsilon_t) = 0$ and $\mathrm{Var}(\upsilon_t) = 1$, a process ε_t is denominated of CGARCH (*p*, *q*) if $\varepsilon_t = \sigma_t \upsilon_t$, where σ_t^2 represents the conditional variance and is a function of $\{\varepsilon_s, s < t\}$, described by equation:

$$\sigma_t^2 = \omega_t + \sum_{i=1}^{p} \alpha_i \left(\varepsilon_{t-i}^2 - \omega_{t-i} \right) + \sum_{j=1}^{q} \beta_j \left(\sigma_{t-j}^2 - \omega_{t-i} \right), \quad (5)$$

where $\omega_t = \alpha_0 + \rho\omega_{t-1} + \eta\left(\varepsilon_{t-1}^2 - \sigma_{t-1}^2\right)$, $\sum_{i=1}^{p}\alpha_i + \sum_{j=1}^{q}\beta_j < 1$ and $\rho < 1$ are necessary conditions to guarantee the stationarity of the process. It can be noticed that the intercept of the model varies over time and follows a dynamic type of first order autoregressive. See (Engle, Lee, 1993) to check the constraints of the parameters so that the conditional variance is positive.

2.3. Distributions of statistical probability

A statistical probability distribution can be summarized from its characteristics, known as distribution moments. The most popular conditional distribution is Gaussian, known as *Normal Distribution*, which can be described by its first two moments: its mean and its variance (Walck, 2007, p. 119). Formally, the probability density function (p.d.f.) of a random variable *x* following this distribution can be expressed by the notation $x \sim N\left(\mu, \sigma^2\right)$ and defined as:

$$f\left(x \middle| \mu, \sigma\right) = \frac{e^{-0.5(x-\mu)^2\sigma^{-2}}}{\sigma\sqrt{2\pi}}, \quad x \in \left(-\infty, \infty\right), \quad \mu \in \left(-\infty, \infty\right), \; \sigma > 0, \tag{6}$$

where μ represents the mean, σ^2 is the variance. In this sense, a random variable with $\mu = 0$ and $\sigma = 1$ is denominated of normal random variable. The assumption of symmetry for the data may cause erroneous inferences about the parameters of interest. In this sense, in order to minimize problems due to the asymmetry of data, Azzalini (1985) proposed the *Skew Normal Distribution* as a generalization of the normal model. Formally, the p.d.f. of a random variable *x* that follows this distribution may be expressed by the notation $x \sim SN\left(0,1,k\right)$ and defined as: $f(x \mid 0,1,k) = 2\varphi(x)\Phi(kx)$, where x and $k \in \mathbb{R}$, $\varphi(\cdot)$ represents the standard normal density function, $\Phi(\cdot)$ exposes the normal distribution function and *k* expresses the asymmetry parameter.

Another widely used distribution in the statistical field is *Student t-distribution*. The characteristics of this distribution are similar to those of the Gaussian distribution, but they have heavier tails, allowing the use of more extreme values. Let Z and S be independent random variables such that $Z \sim N\left(0,1\right)$ and $nS^2 \sim \chi_n^2$, the distribution of $t = Z/S$ is called Student *t*-distribution with n degrees of freedom (Krishnamoorthy, 2006, p. 171). Thus, the p.d.f. of a random variable x following this distribution with n degrees of freedom can be expressed by the notation $x \sim t_n$ and defined as:

$$f\left(x \middle| n\right) = \frac{\Gamma\left(\left(n+1\right)/2\right)}{\Gamma\left(n/2\right)\sqrt{n\pi}}\left(1 + x^2/n\right)^{-(n+1)/2}, \quad x \in \left(-\infty, \infty\right), \; n \geq 1, \tag{7}$$

where n represents a positive integer number, and $\Gamma(\cdot)$ represents the gamma-function defined by $\Gamma\left(x\right) = \int_0^\infty z^{x-1}e^{-z}dz$. To minimize the problems due to data asymmetry, Branco and Dey (2001) developed the *Skew Student t-Distribution*. The use of this distribution is shown to be more robust and adequate than the Skew Normal, since it allows to regulate both the asymmetry and the kurtosis. Being Z and S independent random variables, such that $Z \sim N\left(0,1,k\right)$ and $nS^2 \sim \chi_n^2$, the distribution of $t = Z/\sqrt{S/n}$ is called of Skew Student t-distribution with n degrees of freedom.

Thus, the p.d.f. of such random variable x may be expressed by the notation $x \sim SStd(k,\xi)$ and defined as: $f(x|k,\xi) = 2t_n(x)T_{n+1}\left(kx\sqrt{(1+\xi)/(\xi+x^2)}\right)$, $x \in (-\infty,\infty)$, $n \geq 1$, where $t_n(\cdot)$ represents the p.d.f. of Student distribution with n degrees of freedom, $T_{n+1}(\cdot)$ the cumulative distribution function of Student distribution with $n+1$ degrees of freedom, k expresses the asymmetry parameter, and ξ the kurtosis parameter.

Another possibility of distribution is the *Generalized Error Distribution*, introduced by Subbotin (1923), and that belongs to an exponential family. The conditional density of this distribution is defined by three moments: $\mu \in (-\infty,\infty)$, which determines the position of the curve with respect to the origin, $\sigma \in (0,\infty)$, which determines the dispersion of the distribution curve, and $k \in (0,\infty)$, which controls the asymmetry degree. As such, the p.d.f. of a random variable x that follows the *Generalized Error Distribution* may be expressed by the notation $x \sim G(\mu,\sigma\ ,k)$ and being defined by:

$$f(x|\mu,\sigma,k) = \frac{1}{2^{k+1}\sigma\Gamma(k+1)}\exp\left(-\frac{1}{2}\left|\frac{x-\mu}{\sigma}\right|^{1/k}\right). \tag{8}$$

If $k = 1/2$, equation (8) represents a p.d.f. of normal distribution, meaning, $G(\mu,\sigma^2,1/2) = N(\mu,\sigma^2)$. If $k=1$, equation (8) represents a p.d.f. of a Laplace distribution $L(\mu,4\sigma^2)$. In the limit, if $k \to 0$ the p.d.f. tends to a uniform distribution $U(\mu-\sigma,\mu+\sigma)$. In the same way as skew normal and Student distributions, the *Skew Generalized Error Distribution*, introduced by Theodossiou (1998), respects to an extension of the Generalized Error Distribution that intends to regulate the asymmetry and the kurtosis of the data. Formally the p.d.f. of a random variable x which follows this distribution may be expressed by the notation $x \sim SG(\mu,\sigma^2,k,\xi)$ and defined as

$$f(x|\mu,\sigma,k,\xi) = \frac{C}{\sigma}\exp\left(-\frac{|x-\mu+\psi\sigma|^{\xi}}{\left[1-sign(x-\mu+\psi\sigma)k\right]^{\xi}\theta^{\xi}\sigma^{\xi}}\right),$$

where $C = \frac{\xi}{2\theta}\Gamma(1/\xi)^{-1}$, $\theta = \Gamma(1/\xi)^{1/2}\Gamma(3/\xi)^{-1/2}S(k)^{-1}$, $\psi = 2kAS(k)^{-1}$, $S(k) = \sqrt{1+3k^2-4A^2k^2}$,

$A = \Gamma(2/\xi)\Gamma(1/\xi)^{-1/2}\Gamma(3/\xi)^{-1/2}$, μ represents the mean, σ represents the variance, $k \in (0,\infty)$ controls the asymmetry degree, ξ controls the kurtosis, it is, the height and tails of the density function, $sign$ is the sign function and $\Gamma(\cdot)$ represents gamma-function.

The *Johnson's Su Distribution*, developed by Johnson (1949), may be obtained from the transformation of a standard normal random variable. Formally, the p.d.f. of a random variable x which follows this distribution may be expressed by the notation $x \sim Jsu(\mu,\sigma^2,k,\xi)$ and defined as:

$$f(x|\mu,\sigma,k,\xi) = \frac{\xi}{\sigma\sqrt{1+(x-\mu)^2\sigma^{-2}}}\varphi\left(k+\xi\sinh^{-1}\left(\frac{x-\mu}{\sigma}\right)\right), \quad \sigma > 0, \tag{9}$$

where $\sinh^{-1}$ is the inverse of the hyperbolic sin function, $\varphi(\cdot)$ is the standard normal p.d.f., and ξ must be interpreted as a parameter of kurtosis (positive).

Finally, the eight distribution respects to the *Generalized Hyperbolic Distribution*, introduced by Barndorff-Nielsen and Halgreen (1977). Formally, the p.d.f. of a random variable *x* which follows this distribution may be expressed by the notation $x \sim Ghyp\left(\mu,\sigma^2,k,\xi,\beta\right)$ and defined as:

$$f\left(x|\mu,\sigma,k,\xi,\beta\right)= \\ =a\left(\mu,\sigma,k,\xi,\beta\right)\left[\sigma^2+\left(x-\mu\right)^2\right]^{1/2\xi-1/4} B\left(\xi-0.5,\beta\sqrt{\sigma^2+x^2-2x\mu+\mu^2}\right)e^{k(x-\mu)}, \quad (10)$$

where $a\left(\mu,\sigma,k,\xi,\beta\right)=(\beta^2-k^2)^{1/2\xi}\Big/\sqrt{2\pi\beta^{\xi-1/2}\sigma^{\xi}B\left(\xi,\sigma\sqrt{\beta^2-k^2}\right)}$, $B(\xi,\cdot)$ represents the Bessel modified function of third species with index ξ, and $x\in\mathbb{R}$. The domain of parameters change is represented by $\mu\in\mathbb{R}$, as in: $\sigma\geq 0$ and $|k|<\beta$ if $\xi>0$; $\sigma>0$ and $|k|<\beta$ if $\xi=0$, $\sigma>0$ and $|k|\leq\beta$ if $\xi<0$.

3. Methodology

3.1. Data

The time series used refers to four commodity futures contracts, namely: Crude Oil WTI (Oil), Natural Gas (Gas), Corn (Corn) and Soybean Oil (Soy). The first two refer to non-renewable energy and its contracts are traded on the New York Mercantile Exchange (NYMEX). The latter two relate to raw materials used in the production of biofuels and their contracts are traded on the Chicago Board of Trade (CBOT). All data was collected on the site www.investing.com and refer to daily quotes, in US dollars, of the closing prices of commodities. All sampling periods begin on January 1, 1990 and end on October 31, 2018. In order to minimize the problems that tendencies, seasonality's, influential points and non-linearity provoke, the return of each commodity was calculated by means of the first difference of the logarithms of their prices: $r_t=\left[\ln\left(P_t\right)-\ln\left(P_{t-1}\right)\right]\times 100$, where P_t represents the closing price of the commodity at time *t*.

To verify the presence of structural breaks in commodity prices, the following statistical tests are performed: *Cumulative Sum Control Chart* (CUSUM), developed by Brown et al. (1975); *Moving Sums of Residuals* (MOSUM), proposed by Bauer and Hackl (1978); *supF*, developed by Andrews (1993); *aveF* and *expF*, proposed by Andrews and Ploberger (1994). In order to verify if the series are generated by stationary stochastic processes, the following tests were used: *Augmented Dickey–Fuller* (ADF), developed by Dickey and Fuller (1981); and *Kwiatkowski–Phillips–Schmidt–Shin* (KPSS), developed by Kwiatkowski et al. (1992). Because they were not the object of study of this work, the mathematical formulas of these tests were omitted.

3.2. Method

Each *commodity* returns series have *T* observations and are represented by $X_1,X_2,\ldots,X_T$. All-time series were divided into two subsets: $\left\{X_1,X_2,\ldots,X_n\right\}$ and $\left\{X_{n+1},X_{n+2},\ldots,X_T\right\}$, where *n* represents the start of the forecasting. The first subset represents the in-sample period, where the parameters of the competing models are estimated. The second subset represents the period called out-of-sample, in which the forecasts of the competing models are realized and evaluated. After dividing the sample, the out-of-sample evaluation and forecasting process begins with the following steps.

Define $z = n$ to be the origin point of the forecasting. Soon after, the parameters of the competing models are estimated using the subset $\{X_1, X_2, \ldots, X_z\}$.

One-step-ahead prediction is performed for each competing model, using its parameters estimated in step 1.

The origin of the forecasting is increased in one observation ($z = z + 1$) and we restart the process in step 1. This procedure is repeated until we reach the last available observation of the time series, where the forecasting origin *z* equals point *T*. In this way, the model parameters are estimated in a moving window size that adjusts to each step.

Once the prediction process is defined, forecasts for 100 steps ahead with parameter readjustment at each step for all competing models are performed. The choice of out-of-sample size was established arbitrarily, justifying itself by representing a sufficient number, without harming the period of estimation of the parameters, to measure the predictive performance of the models. The total number of estimated models was set at 40, which results in a matrix with a total of 4000 predictions estimated VaR forecasts for each commodity at a 1% confidence level. This results in 16000 forecasts. With this data it is possible to calculate the VaR loss associated with each model at each point in time. Since VaR can be defined as the conditional quantile $\Pr\left(r_t \le \mathrm{VaR}_t(\alpha)\right) = \alpha$, the measurement of VaR loss for each forecast obtained is performed by $\ell\left(r_t, \mathrm{VaR}_t(\alpha)\right) = \left(\alpha - d_t(\alpha)\right)\left(r_t - \mathrm{VaR}_t(\alpha)\right)$, where $d_t(\alpha) = \mathbf{1}\left(r_t < \mathrm{VaR}_t(\alpha)\right)$, is the α-quantile loss function (for more details, see (González-Rivera et al., 2004)). Now to compare the forecasts obtained by the various alternative model specifications, the Model Confidence Set (MCS) procedure, proposed by Hansen et al. (2011), and three different measures of accuracy are used, namely Mean Squared Error (MSE), Root Mean Square Error (RMSE), suggested by Bollerslev et al. (1994), and Mean Absolute Deviation (MAD), suggested by Hansen and Lunde (2005). Because they are not object of study of this work, the mathematical formulas of these functions have been omitted, but they can be consulted in the references presented.

4. Empirical evidence and discussion

When specifying an econometric model, it is assumed that its assumptions apply to all observations in the sample data. The parameters estimated by the model must be the same for all the subsets of the sample, that is, the time series cannot present structural changes in the relation between the parameters estimated by the model. Thus, this study applied five different statistical tests to verify and identify the presence of structural changes in the analyzed series. With the results of these tests, it was possible to find periods when energy commodity prices show no structural changes over time. Table 1 shows the most recent periods found and that were used as a basis for the estimation of the models proposed in this study.

The descriptive statistics of the four-time series are shown in Table 2. Both series have leptokurtic distributions and with tails heavier than a Gaussian distribution, since they exhibit asymmetry and excesses of kurtosis. These evidence of non-normality in probability density functions are also confirmed by the Jarque–Bera test, which rejects the null hypothesis of normality for all

Table 1. Exposition of the analyzed period and total number of observations for each series

Time series	Time period		Number of observations
	Start	End	
Oil	08/09/2014	31/10/2018	1094
Gas	19/09/2014	31/10/2018	1084
Corn	09/07/2014	31/10/2018	1117
Soy	27/06/2014	31/10/2018	1127

Table 2. Series descriptive statistics for the identified period after the last structural break

Series	Obs.	Mean	Median	Maximum	Minimum	Stdv	Skewness	Kurtosis	JB
				In-sample					
Oil	994	52.24	49.68	94.88	26.21	11.73	1.10	5.03	369.65
Gas	984	2.83	2.83	4.49	1.64	0.49	0.31	4.00	56.43
Corn	1017	366.34	365.25	437.75	301.50	21.25	0.37	3.60	38.80
Soy	1027	32.29	32.30	38.87	26.05	2.11	–0.21	3.79	34.34
				Out-of-sample					
Oil	100	69.71	69.17	76.41	65.01	2.74	0.43	2.41	4.50
Gas	100	2.95	2.93	3.32	2.72	0.16	0.64	2.28	8.93
Corn	100	357.64	359.50	378.25	330.25	11.11	–0.45	2.32	5.32
Soy	100	28.44	28.34	29.76	27.12	0.54	0.24	2.77	1.18

Notes. Stdv — standard deviation, JB — Jarque–Bera test.

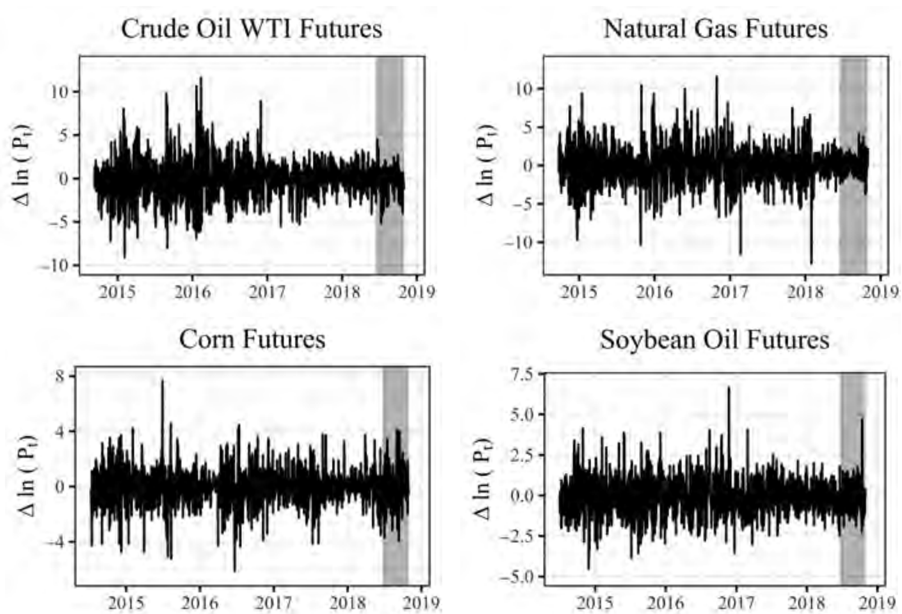

Fig. 1. Log returns of price for the sample period.
The highlighted period (grey) is the out-of-sample period

four energy commodities. These characteristics show that the use of position, scaling and shape parameters may be more adequate for the estimation of the probability density function of the time series analyzed.

The graphical representations of the first difference of logarithms of the prices of the four energy commodities are shown in Figure 1. It can be seen that the behavior of the evolution of the returns of the assets diverges between the time series analyzed. Some considerations can be drawn from this graphical analysis, namely: (a) the returns of non-renewable energy commodities have higher extreme values than those of renewable energies; (b) the variability seems to be smaller in the Oil series and higher in the Gas series; and (c) the observations of the four series appear to return to their mean and the fluctuations around the mean seem to exhibit a more or less constant amplitude, showing that the series may be behaving as a second order stationary process.

Stationarity plays a central role in the analysis of time series, since it represents a natural way to replace the hypothesis that observations are a sequence of independent and identically distributed random variables. Therefore, in order to certify that the series are generated by stationary stochastic processes, the ADF and KPSS tests were applied. The results obtained by the tests are presented in Table 3 and show that all series have a unitary integration order, that is, they all follow a stochastic stationary process only in their first difference of logarithms, with a 1% level of significance.

After the analysis of the return series' behavior was completed, comparison of the predictive capacity of different heteroscedastic models was performed using procedure described before. Table 4 summarizes the five best results obtained for the four energy commodities, ranked in ascending order of the estimated values of the RMSE measure statistic. The evidence found for all accuracy measures for all competing models can be found in Appendix A.

The results achieved are diverse and can be analyzed in several ways. However, some important points should be mentioned.

(a) The hypothesis that the most recent heteroscedastic models, which incorporate several parameters in their mathematical expressions, have a predictive capacity of conditional variance superior to the older and simplified models, is invalid for most of the analyzed series. The CGARCH model, which is the most sophisticated used in this study, only performed regularly in estimating the volatility of the Gas commodity. In part, this evidence is at odds with the results found by Bentes (2015); Bui Quang et al. (2018); Klein, Walther (2016); Zhang et al. (2018b), which show

Table 3. Statistics of the unit root tests for the analyzed series

Series	ADF		KPSS	
	Statistic	*P*-value	Statistic	*P*-value
Oil	–3.87	0.0156	2.81	< 0.0100
Δln (Oil)	–9.74	< 0.0100	0.41	0.07249
Gas	–2.97	0.1663	1.28	< 0.0100
Δln (Gas)	–10.70	< 0.0100	0.13	> 0.1000
Corn	–4.55	< 0.0100	0.85	< 0.0100
Δln (Corn)	–9.70	< 0.0100	0.03	> 0.1000
Soy	–2.68	0.2888	1.12	< 0.0100
Δln (Soy)	–11.73	< 0.0100	0.08	> 0.1000

Notes. ADF — Augmented Dickey–Fuller test, KPSS — Kwiatkowski–Phillips–Schmidt–Shin test.

Table 4. Summary of results obtained using the three loss functions

Series/Models	RMSE		MSE		MAD	
	Statistic	Position	Statistic	Position	Statistic	Position
Oil						
GARCH-Ghyp	0.091376	1	0.008350	1	0.053668	6
GARCH-Jsu	0.091624	2	0.008395	2	0.053635	5
GARCH-SGed	0.091928	3	0.008451	3	0.053730	8
GARCH-SStd	0.093679	4	0.008776	4	0.053479	3
GARCH-Ged	0.110696	5	0.012254	5	0.053288	2
Gas						
GJR-GARCH-SNorm	0.042867	1	0.001838	1	0.039620	1
CGARCH-SNorm	0.042927	2	0.001843	2	0.039728	2
GARCH-SNorm	0.043075	3	0.001855	3	0.039850	3
GJR-GARCH-Norm	0.043285	4	0.001874	4	0.040044	4
CGARCH-Norm	0.043368	5	0.001881	5	0.040175	5
Corn						
EGARCH-Ged	0.042822	1	0.001834	1	0.039249	1
APARCH-Ged	0.043209	2	0.001867	2	0.039593	2
EGARCH-Std	0.043216	3	0.001868	3	0.039622	3
APARCH-Std	0.043577	4	0.001899	4	0.040039	5
EGARCH-SGed	0.043599	5	0.001901	5	0.040030	4
Soy						
EGARCH-Norm	0.024979	1	0.000624	1	0.022649	2
GARCH-Jsu	0.025083	2	0.000629	2	0.022905	5
GARCH-Ghyp	0.025088	3	0.000629	3	0.022904	4
EGARCH-SNorm	0.025176	4	0.000634	4	0.022552	1
EGARCH-SGed	0.025181	5	0.000634	5	0.022875	3

Note. In this table and also in Tables A1–A4, B1, B2 in Appendices we use the following designations for distributions from section 2.3:
Norm — *Normal Distribution,* SNorm — *Skew Normal Distribution,* Std — *Student t-distribution,*
SStd — *Skew Student t-distribution,* Ged — *Generalized Error Distribution,*
SGed — *Skew Generalized Error Distribution,* Jsu — *Johnson's Su Distribution,*
Ghyp — *Generalized Hyperbolic Distribution*.

that the incorporation of parameters into the mathematical structure of the heteroscedastic model improves the estimation of the conditional variance of the analyzed time series. The results of these studies are notable for statistical modeling, but are limited to the use of probability distributions with simplified structures, such as Gaussian and Student, which may underestimate or overestimate the measure of risk measured, as the work of Slim et al. (2017) demonstrates.

(b) The use of different probability density functions proved to be appropriate, since, according to the previous analysis of descriptive statistics, the sample observations of the data do not have equal behavior and, therefore, should be worked out differently. The choice of the correct density function for the data seems to influence the results of the predictions more than the choice of the heteroscedastic model itself. For example, the Oil series has the same heteroscedastic model in the first five positions and the chosen density function directly influences the results obtained by the model. For this commodity, the more complex density functions, which use several parameters in their mathematical structure, have proved to be more adequate than the more simplified functions. Thus, the use of the Gaussian distribution is not adequate and would lead to erroneous predictions of Oil commodity volatility, since the distribution that best describes the series data needs to refer to the position of the curve in relation to the origin, to the dispersion of the curve, degree of asymmetry, kurtosis, degree of elevation or flattening of distribution, and other factors. These considerations can also be extended to the Soy series.

Normal and asymmetric normal distributions produced the best volatility forecasts for gas commodity returns. That shows that the mean, variance, and asymmetry are the distribution moments sufficient to model the natural gas commodity returns real data. Finally, asymmetric distributions that have heavier tails and allow for more extreme values in the sample data led to the best Corn price volatility predictions. A similar result was found by Hasanov et al. (2018), which showed that each of the six energy commodities analyzed by the author was better fitted by a given density function. All this evidence supports the use of alternative probability distributions in econometric models. In this particular research, energy commodities fitted better to distributions with more sophisticated mathematical structures than the normal distribution. This fact is relevant and in line with the evidence found by Gunay, Khaki (2018); Lyu et al. (2017); Slim et al. (2017).

(c) The hypothesis of the existence of a single model superior to all others for all energy commodities proved to be inadequate. For the commodity oil price volatility, the simple GARCH model proved to be superior to other heteroscedastic models, showing that the conditional variance expressed as a linear function of the square of past errors and its own past values is sufficient and preferable for a good VaR estimation.

The Soy series also seems to be better modelled by simple heteroscedastic models, especially EGARCH, which is a small extension of the GARCH, differing only by considering that positive and negative shocks do not have the same impact on conditional variance. Volatility of Gas and Corn commodities was better modeled by more flexible heteroscedastic models, such as APARCH and CGARCH. These results are in agreement with the evidence found by Bui Quang et al. (2018); Chu et al. (2017); Hasanov et al. (2018); Slim et al. (2017), and demonstrate that the choice of a universal model for any time series is not adequate.

(d) Finally, heteroscedastic models with more simplified structures seem to be better adapted to the probability density functions with more complex structures and vice versa. Therefore, the hypothesis that the union of complex heteroscedastic models with mathematically complex density functions results in better VaR measurements seems to be wrong.

Finally, as a complementary analysis, the MCS procedure is performed. The central idea is to provide a set of superior models in which the null hypothesis of equal predictive ability cannot be rejected at a given level of statistical significance. In other words, all the models presented in this set have statistically the same predictive ability and excel over their competitors. The results of the MCS procedure are presented in Appendix B and some considerations can be pointed out, namely:

(a) The hypothesis that heteroscedastic models with more complex structures are superior to those with simpler structures has once again been shown to be invalid. Confirming the results found previously. For estimating Oil commodity volatility both the simpler GARCH model and the more complex CGARCH model are superior to the others and have statistically the same predictive ability. A similar result can be seen for the commodity Soy, which shows the GARCH, EGARCH and CGARCH models as preferable to the other models. For Corn commodity volatility, on the other hand, using extreme heteroscedastic models, such as GARCH and CGARCH, does not seem to be appropriate. In this case, intermediate models are preferable. Finally, for Gas commodity volatility no specific heteroscedastic model proved superior to its competitors.

(b) The use of different probability density functions proved to be appropriate for some commodities. Confirming again the previous results. For the estimates of commodity Gas risk measures, choosing the correct statistical probability distribution seems to be more important than choosing the heteroscedastic model to be used. In this case, simpler distributions, such as the normal and the asymmetric normal, proved to be preferable to the others. For commodity Corn, on the other hand, distributions that allow for heavier tails, such as Student and General Error distribution, are better suited to be used. These results are quite similar to those found previously. Now, for the commodities Oil and Soy, the results found were a little different from the previous ones. For these commodities, the choice of probability distribution does not seem to influence the performance of the risk measure estimates as much, since both the simpler and more complex distributions have statistically similar results.

(c) The hypothesis of the existence of a single model superior to all others to measure risk measures for all energy commodities has also proven inadequate. Therefore, although at some points there may be slight differences, the MCS procedure helped to confirm and strengthen the central results found earlier.

5. Conclusions

Research involving the estimation and prediction of asset volatility is widely accepted in the academic and financial areas, as it can help optimize investment portfolios, hedge, price options, measure risks, and so on. Understanding asset price behavior is essential for companies to make decisions about how much and where they should invest their resources in order to maximize the likelihood of the relationship between loss and gain in their investments. Greater efficiency in planning on the feasibility of applying resources can provide an increase in the profit margins of the companies, making possible new investments and making them more competitive in the market in which they are inserted. In this context, energy commodity futures contracts have attracted the attention of several economic agents, since they can serve both for speculative purposes and for protection purposes in the real economy. For these and other reasons, it is essential to model the behavior of energy commodity prices and find appropriate tools to quantify the risks inherent in the market.

The importance of accuracy in estimating the volatility of financial assets has meant that ARCH family models and their forms of application have evolved significantly over the last few years. Models with complex mathematical structures have been developed along with procedures that can help improve the estimation of volatility, such as the use of different distributions of statistical probability. Thus, the objective of this study was to compare the predictive capacity of the conditional variance of different models by measuring the VaR of four futures contracts of energy commodities and to try to find patterns of behavior or indications of the best use of these models.

As a result, some considerations can be exposed.

(a) There is no single model that can be used in a generalized way to measure the VaR of the different energy commodities. Each time series has a certain behavior, and the use of a single model would lead to erroneous measurements of conditional variance and VaR. Therefore, this study does not provide a general model but a set of procedures that can be adopted to measure risk measures more adequately.

(b) Although there are a significant number of probability distributions available in the academic literature (see, for example, (Azzalini, 1985; Barndorff-Nielsen, Halgreen, 1977; Branco, Dey, 2001; Johnson, 1949; Subbotin, 1923; Theodossiou, 1998)), few studies make use of different Gaussian distributions in the estimation of volatility by heteroscedastic models, and this work showed that the use of different density functions is extremely important in the measurement of the conditional variance of time series. In a sense, the choice of a suitable density function may be more relevant than the choice of an alternative heteroscedastic model in the VaR estimation of energy commodities. The Ardia et al. (2019), Caporale, Zekokh (2019), Maciel (2021) and Wu et al. (2020) studies also confirm this evidence in other data sets.

(c) The incorporation of several additional parameters into the mathematical expression of the heteroscedastic models and the density functions does not always improve the predictive capacity of the conditional variance when used in the VaR measurement. Each time series is composed of a number of sample observations that have different behaviors and these behaviors must be taken into account in the estimations of the econometric models. For some data samples, few reference parameters may be sufficient to describe their characteristics. For other time series, a greater number of parameters may be more appropriate. The models used must adapt to the data and not the other way around.

The evidence found may help agents to improve their ways of quantifying investment risks as this study alerts some important questions that should be considered in the use of heteroskedastic models and risk measures. Finally, we propose suggestions for future research, namely: (a) estimate VaR using different time horizons, in order to verify the robustness of the results found in this work; it is proposed to estimate the VaR for 50 or 150 steps ahead, with readjustment of the parameters at each step; (b) verify if the non-verification of structural changes in the time series actually modify the results found in this study, since the combination of heteroscedastic models with different density functions may be powerful enough to absorb the structural changes that occur in the time series; (c) verifying if indeed the conditional variances are strongly persistent, especially for long series, as the traditional theory shows. This suggestion is relevant since some recent studies have shown that structural changes in the dynamics of volatility can occur over time and, if they occur, fixed-parameter heteroscedastic models may fail in their attempt to capture the true volatility variation (Ardia et al., 2018; Bauwens et al., 2014; Maciel, 2021; Siu, 2021; Tan et al., 2021; Wang et al., 2021; Wu et al., 2020).

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Appendix A.
Results obtained

Table A1. Results achieved by three loss functions for the Crude Oil WTI (Oil) commodity

Models	RMSE		MSE		MAD	
	Statistic	Position	Statistic	Position	Statistic	Position
GARCH-Ghyp	0.091376	1	0.008350	1	0.053668	6
GARCH-Jsu	0.091624	2	0.008395	2	0.053635	5
GARCH-SGed	0.091928	3	0.008451	3	0.053730	8
GARCH-SStd	0.093679	4	0.008776	4	0.053479	3
GARCH-Ged	0.110696	5	0.012254	5	0.053288	2
GARCH-Std	0.110719	6	0.012259	6	0.053083	1
GARCH-SNorm	0.116644	7	0.013606	7	0.053606	4
CGARCH-Jsu	0.120144	8	0.014434	8	0.053785	9
CGARCH-Ghyp	0.121984	9	0.014880	9	0.054133	11
CGARCH-SStd	0.122164	10	0.014924	10	0.054123	10
CGARCH-SGed	0.127661	11	0.016297	11	0.055089	12
GARCH-Norm	0.127761	12	0.016323	12	0.053699	7
CGARCH-Std	0.139393	13	0.019431	13	0.055612	13
CGARCH-Ged	0.157406	14	0.024777	14	0.057741	15
GJR-GARCH-Jsu	0.161023	15	0.025928	15	0.061628	32
GJR-GARCH-Ghyp	0.161125	16	0.025961	16	0.061755	33
GJR-GARCH-SGed	0.162226	17	0.026317	17	0.061798	34
GJR-GARCH-SStd	0.162766	18	0.026493	18	0.061379	28
APARCH-SStd	0.163745	19	0.026812	19	0.060499	20
CGARCH-SNorm	0.163763	20	0.026818	20	0.057355	14
APARCH-Ghyp	0.169665	21	0.028786	21	0.060186	17
APARCH-SGed	0.173533	22	0.030114	22	0.060771	23
CGARCH-Norm	0.176400	23	0.031117	23	0.058529	16
EGARCH-Ghyp	0.178054	24	0.031703	24	0.060371	18
EGARCH-Jsu	0.178707	25	0.031936	25	0.061032	26
EGARCH-SGed	0.179435	26	0.032197	26	0.060444	19
GJR-GARCH-Std	0.179809	27	0.032331	27	0.060798	24
GJR-GARCH-SNorm	0.180027	28	0.032410	28	0.060997	25
EGARCH-SStd	0.180846	29	0.032705	29	0.062010	35
APARCH-Jsu	0.181615	30	0.032984	30	0.060724	22
GJR-GARCH-Ged	0.181680	31	0.033008	31	0.061079	27
APARCH-Ged	0.187698	32	0.035230	32	0.064162	37
APARCH-SNorm	0.188149	33	0.035400	33	0.065230	38
GJR-GARCH-Norm	0.193176	34	0.037317	34	0.060505	21
APARCH-Std	0.193357	35	0.037387	35	0.065403	39
EGARCH-Std	0.194705	36	0.037910	36	0.061416	29

End of Table A1

Models	RMSE		MSE		MAD	
	Statistic	Position	Statistic	Position	Statistic	Position
EGARCH-SNorm	0.195177	37	0.038094	37	0.061433	30
EGARCH-Ged	0.195685	38	0.038293	38	0.061548	31
EGARCH-Norm	0.208214	39	0.043353	39	0.062818	36
APARCH-Norm	0.209781	40	0.044008	40	0.069088	40

Table A2. Results achieved by three loss functions for the Natural Gas (Gas) commodity

Models	RMSE		MSE		MAD	
	Statistic	Position	Statistic	Position	Statistic	Position
GJR-GARCH-SNorm	0.042867	1	0.001838	1	0.039620	1
CGARCH-SNorm	0.042927	2	0.001843	2	0.039728	2
GARCH-SNorm	0.043075	3	0.001855	3	0.039850	3
GJR-GARCH-Norm	0.043285	4	0.001874	4	0.040044	4
CGARCH-Norm	0.043368	5	0.001881	5	0.040175	5
GARCH-Norm	0.043506	6	0.001893	6	0.040288	6
EGARCH-SNorm	0.043626	7	0.001903	7	0.040413	7
APARCH-SNorm	0.043887	8	0.001926	8	0.040715	8
EGARCH-Norm	0.044056	9	0.001941	9	0.040851	9
APARCH-Norm	0.044365	10	0.001968	10	0.041203	10
CGARCH-SStd	0.045085	11	0.002033	11	0.042027	11
GARCH-SStd	0.045120	12	0.002036	12	0.042053	12
CGARCH-Jsu	0.045123	13	0.002036	13	0.042066	13
GARCH-Jsu	0.045151	14	0.002039	14	0.042084	14
CGARCH-Ghyp	0.045257	15	0.002048	15	0.042207	15
GARCH-Ghyp	0.045292	16	0.002051	16	0.042234	16
GJR-GARCH-Jsu	0.045302	17	0.002052	17	0.042241	18
GJR-GARCH-SStd	0.045303	18	0.002052	18	0.042240	17
GJR-GARCH-Ghyp	0.045475	19	0.002068	19	0.042421	21
EGARCH-Jsu	0.045499	20	0.002070	20	0.042391	19
EGARCH-SStd	0.045514	21	0.002072	21	0.042407	20
GJR-GARCH-SGed	0.045588	22	0.002078	22	0.042540	23
EGARCH-Ghyp	0.045596	23	0.002079	23	0.042489	22
CGARCH-SGed	0.045603	24	0.002080	24	0.042575	25
GARCH-SGed	0.045613	25	0.002081	25	0.042565	24
APARCH-SStd	0.045802	26	0.002098	26	0.042731	26
APARCH-Jsu	0.045825	27	0.002100	27	0.042758	27
APARCH-Ghyp	0.045913	28	0.002108	28	0.042845	29
EGARCH-SGed	0.045918	29	0.002108	29	0.042841	28
GJR-GARCH-Ged	0.046324	30	0.002146	30	0.043293	30
CGARCH-Ged	0.046345	31	0.002148	31	0.043334	32
APARCH-SGed	0.046346	32	0.002148	32	0.043320	31

End of Table A2

Models	RMSE		MSE		MAD	
	Statistic	Position	Statistic	Position	Statistic	Position
GARCH-Ged	0.046367	33	0.002150	33	0.043337	33
CGARCH-Std	0.046473	34	0.002160	34	0.043463	34
GARCH-Std	0.046524	35	0.002164	35	0.043508	35
EGARCH-Ged	0.046628	36	0.002174	36	0.043565	36
GJR-GARCH-Std	0.046776	37	0.002188	37	0.043765	37
EGARCH-Std	0.046882	38	0.002198	38	0.043817	38
APARCH-Ged	0.047119	39	0.002220	39	0.044111	39
APARCH-Std	0.047272	40	0.002235	40	0.044249	40

Table A3. Results achieved by three loss functions used for the Corn (Corn) commodity

Models	RMSE		MSE		MAD	
	Statistic	Position	Statistic	Position	Statistic	Position
EGARCH-Ged	0.042822	1	0.001834	1	0.039249	1
APARCH-Ged	0.043209	2	0.001867	2	0.039593	2
EGARCH-Std	0.043216	3	0.001868	3	0.039622	3
APARCH-Std	0.043577	4	0.001899	4	0.040039	5
EGARCH-SGed	0.043599	5	0.001901	5	0.040030	4
GARCH-Ged	0.043609	6	0.001902	6	0.040061	6
GJR-GARCH-Ged	0.043696	7	0.001909	7	0.040096	7
CGARCH-Ged	0.043997	8	0.001936	8	0.040546	10
APARCH-SGed	0.044053	9	0.001941	9	0.040430	8
GARCH-Std	0.044054	10	0.001941	10	0.040560	11
GJR-GARCH-Std	0.044095	11	0.001944	11	0.040540	9
CGARCH-Std	0.044108	12	0.001945	12	0.040700	12
GARCH-SGed	0.044211	13	0.001955	13	0.040705	13
EGARCH-Ghyp	0.044282	14	0.001961	14	0.040758	15
EGARCH-SStd	0.044286	15	0.001961	15	0.040754	14
CGARCH-SGed	0.044320	16	0.001964	16	0.040896	18
EGARCH-Jsu	0.044348	17	0.001967	17	0.040824	16
GJR-GARCH-SGed	0.044400	18	0.001971	18	0.040841	17
CGARCH-Ghyp	0.044589	19	0.001988	19	0.041205	20
CGARCH-Jsu	0.044721	20	0.002000	20	0.041355	23
CGARCH-SStd	0.044729	21	0.002001	21	0.041367	25
APARCH-SStd	0.044733	22	0.002001	22	0.041148	19
APARCH-Ghyp	0.044798	23	0.002007	23	0.041223	21
GARCH-SStd	0.044831	24	0.002010	24	0.041394	26
APARCH-Jsu	0.044845	25	0.002011	25	0.041271	22
GARCH-Ghyp	0.044847	26	0.002011	26	0.041402	27
GARCH-Jsu	0.044880	27	0.002014	27	0.041444	28
GJR-GARCH-SStd	0.045053	28	0.002030	28	0.041557	31

End of Table A3

Models	RMSE		MSE		MAD	
	Statistic	Position	Statistic	Position	Statistic	Position
GJR-GARCH-Ghyp	0.045103	29	0.002034	29	0.041607	33
GJR-GARCH-Jsu	0.045151	30	0.002039	30	0.041661	35
GJR-GARCH-SNorm	0.052961	31	0.002805	31	0.041358	24
CGARCH-Norm	0.053625	32	0.002876	32	0.041557	30
CGARCH-SNorm	0.053801	33	0.002895	33	0.041622	34
GARCH-SNorm	0.054100	34	0.002927	34	0.041527	29
APARCH-SNorm	0.055144	35	0.003041	35	0.041728	37
EGARCH-SNorm	0.055750	36	0.003108	36	0.041583	32
GJR-GARCH-Norm	0.055858	37	0.003120	37	0.041675	36
GARCH-Norm	0.056620	38	0.003206	38	0.041799	38
EGARCH-Norm	0.059457	39	0.003535	39	0.041951	39
APARCH-Norm	0.059476	40	0.003537	40	0.042188	40

Table A4. Results achieved by three loss functions used for the Soybean Oil (Soy) commodity

Models	RMSE		MSE		MAD	
	Statistic	Position	Statistic	Position	Statistic	Position
EGARCH-Norm	0.024979	1	0.000624	1	0.022649	2
GARCH-Jsu	0.025083	2	0.000629	2	0.022905	5
GARCH-Ghyp	0.025088	3	0.000629	3	0.022904	4
EGARCH-SNorm	0.025176	4	0.000634	4	0.022552	1
EGARCH-SGed	0.025181	5	0.000634	5	0.022875	3
GARCH-Norm	0.025190	6	0.000635	6	0.022962	9
GARCH-SGed	0.025217	7	0.000636	7	0.023037	12
CGARCH-Jsu	0.025235	8	0.000637	8	0.022934	7
GARCH-SStd	0.025245	9	0.000637	9	0.022962	10
CGARCH-Ghyp	0.025256	10	0.000638	10	0.022940	8
GJR-GARCH-Jsu	0.025306	11	0.000640	11	0.023149	19
GJR-GARCH-Ghyp	0.025355	12	0.000643	12	0.023205	21
CGARCH-SGed	0.025364	13	0.000643	13	0.023079	15
APARCH-Jsu	0.025439	14	0.000647	14	0.023270	24
CGARCH-Norm	0.025439	15	0.000647	15	0.023038	13
EGARCH-Jsu	0.025446	16	0.000648	16	0.023161	20
CGARCH-SStd	0.025451	17	0.000648	17	0.023067	14
GJR-GARCH-SStd	0.025473	18	0.000649	18	0.023217	22
GJR-GARCH-SGed	0.025511	19	0.000651	19	0.023243	23
GJR-GARCH-Norm	0.025516	20	0.000651	20	0.023141	18
APARCH-Norm	0.025584	21	0.000655	21	0.023291	25
APARCH-Ghyp	0.025631	22	0.000657	22	0.023445	27
EGARCH-Ghyp	0.025636	23	0.000657	23	0.023367	26
APARCH-SGed	0.025666	24	0.000659	24	0.023529	32

End of Table A4

Models	RMSE		MSE		MAD	
	Statistic	Position	Statistic	Position	Statistic	Position
GARCH-Ged	0.025705	25	0.000661	25	0.023460	29
GJR-GARCH-Ged	0.025739	26	0.000663	26	0.023514	30
APARCH-SStd	0.025748	27	0.000663	27	0.023524	31
EGARCH-SStd	0.025802	28	0.000666	28	0.023550	33
CGARCH-Ged	0.025827	29	0.000667	29	0.023455	28
GARCH-Std	0.025849	30	0.000668	30	0.023614	34
EGARCH-Ged	0.025876	31	0.000670	31	0.023637	35
GJR-GARCH-Std	0.026021	32	0.000677	32	0.023817	37
CGARCH-Std	0.026026	33	0.000677	33	0.023680	36
EGARCH-Std	0.026319	34	0.000693	34	0.024117	38
GARCH-SNorm	0.026334	35	0.000694	35	0.022930	6
APARCH-SNorm	0.026376	36	0.000696	36	0.023125	17
APARCH-Ged	0.026504	37	0.000702	37	0.024318	39
APARCH-Std	0.026529	38	0.000704	38	0.024352	40
CGARCH-SNorm	0.026538	39	0.000704	39	0.023021	11
GJR-GARCH-SNorm	0.027128	40	0.000736	40	0.023088	16

Appendix B.
Superior set of models

For more information about the statistics used in the MCS procedure and laid out in this Appendix, see (Hansen et al., 2011).

Table B1. Superior set of models for Crude Oil WTI (Oil) and Natural Gas (Gas)

Models	$\text{Rank}_{R,M}$	t_{ij}	$P\text{–value}_{R,M}$	$\text{Rank}_{MAX,M}$	t_i	$P\text{–value}_{MAX,M}$	$\text{Loss}\times 10^3$
			Oil: 23 eliminations				
GARCH-Norm	1	–5.16	1.00	3	0.35	0.99	53.70
CGARCH-SStd	2	–1.84	1.00	7	0.87	0.93	54.12
CGARCH-Jsu	3	–1.55	1.00	5	0.72	0.95	53.79
GARCH-SNorm	4	–1.35	1.00	6	0.86	0.93	53.61
CGARCH-Ghyp	5	–1.32	1.00	14	1.96	0.26	54.13
GARCH-Std	6	–1.22	1.00	1	–0.21	1.00	53.08
GARCH-Ged	7	–1.09	1.00	17	4.99	0.00	53.29
GARCH-SStd	8	–0.47	1.00	2	0.21	1.00	53.48
GARCH-Jsu	9	–0.40	1.00	4	0.62	0.96	53.64
GARCH-Ghyp	10	–0.39	1.00	9	0.90	0.93	53.67
GARCH-SGed	11	–0.37	1.00	15	2.15	0.17	53.73
CGARCH-SGed	12	–0.11	1.00	16	3.44	0.01	55.09
CGARCH-Std	13	0.22	0.97	8	0.88	0.93	55.61
CGARCH-SNorm	14	0.62	0.88	12	1.06	0.88	57.35

End of Table B1

Models	$Rank_{R,M}$	t_{ij}	$P\text{–}value_{R,M}$	$Rank_{MAX,M}$	t_i	$P\text{–}value_{MAX,M}$	$Loss \times 10^3$
CGARCH-Norm	15	0.69	0.87	11	1.00	0.91	58.53
EGARCH-Norm	16	0.83	0.84	10	1.00	0.92	62.82
CGARCH-Ged	17	0.84	0.83	13	1.32	0.88	57.74
Gas: 34 eliminations							
GJR-GARCH-SNorm	1	–9.23	1.00	1	–1.38	1.00	39.62
CGARCH-SNorm	2	–3.55	1.00	2	1.38	1.00	39.73
GARCH-SNorm	3	–2.33	1.00	4	3.96	0.00	39.85
GJR-GARCH-Norm	4	1.72	0.14	5	15.00	0.00	40.04
EGARCH-SNorm	5	2.38	0.03	3	3.88	1.00	40.41
CGARCH-Norm	6	3.23	0.00	6	15.30	0.00	40.18

Table B2. Superior set of models for Corn (Corn) and Soybean Oil (Soy)

Models	$Rank_{R,M}$	t_{ij}	$P\text{–}value_{R,M}$	$Rank_{MAX,M}$	t_i	$P\text{–}value_{MAX,M}$	$Loss \times 10^3$
Corn: 27 eliminations							
EGARCH-Ged	1	–2.10	1.00	1	–0.70	1.00	39.25
APARCH-Ged	2	–1.40	1.00	5	3.16	0.02	39.59
EGARCH-Std	3	–1.27	1.00	8	5.33	0.00	39.62
APARCH-Std	4	–0.51	1.00	9	6.29	0.00	40.04
EGARCH-SGed	5	–0.50	1.00	13	20.03	0.00	40.03
GARCH-Ged	6	–0.43	1.00	4	2.68	0.11	40.06
GJR-GARCH-Ged	7	–0.39	1.00	6	3.17	0.02	40.10
APARCH-SGed	8	0.27	0.93	12	13.61	0.00	40.43
GJR-GARCH-SNorm	9	0.43	0.84	3	0.71	1.00	41.36
EGARCH-SNorm	10	0.46	0.83	2	0.70	1.00	41.58
CGARCH-Ged	11	0.50	0.80	7	3.83	0.00	40.55
GARCH-Std	12	0.51	0.79	10	7.89	0.00	40.56
GJR-GARCH-Std	13	0.53	0.79	11	8.14	0.00	40.54
Soy: 29 eliminations							
EGARCH-SNorm	1	–1.43	1.00	1	–0.17	1.00	22.55
EGARCH-Norm	2	–0.59	1.00	2	0.17	1.00	22.65
EGARCH-SGed	3	–0.03	1.00	11	3.13	0.01	22.87
GARCH-Ghyp	4	0.07	0.99	5	1.00	0.82	22.90
GARCH-SNorm	5	0.08	0.99	4	0.97	0.83	22.93
GARCH-Jsu	6	0.10	0.99	3	0.89	0.86	22.90
GARCH-SStd	7	0.20	0.93	10	1.65	0.42	22.96
CGARCH-Ghyp	8	0.21	0.93	9	1.22	0.73	22.94
CGARCH-Jsu	9	0.22	0.92	6	1.07	0.80	22.93
CGARCH-SNorm	10	0.23	0.91	7	1.08	0.79	23.02
GJR-GARCH-SNorm	11	0.30	0.87	8	1.10	0.79	23.09