Прикладная эконометрика, 2022, т. 66, с. 68–84.
Applied Econometrics, 2022, v. 66, pp. 68–84.
DOI: 10.22394/1993-7601-2022-66-68-84

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Price multifractality and informational efficiency in the futures markets of the US soybean complex

This work investigates price multifractality and informational efficiency in the futures markets of the US soybean complex (soybeans, soybean meal, soybean oil, and the crush spread) using daily prices from 2015 to 2021 and Multifractal Detrended Fluctuation Analysis (MFDFA). The empirical findings suggest: First, none of the four series exhibited long-range dependence. They did, however, show considerable serial dependence locally. The futures prices of soybeans and soybean oil, and of the crush spread were locally anti-persistent (persistent) for large (small) fluctuations whereas the futures prices of soybean meal were persistent for all small and large fluctuations. Second, all markets in the US soybean complex exhibited some degree of informational inefficiency with that of the crush spread being less efficient relative to the other three. Overall, the results provide valuable information to investors as to whether trend-following or oscillatory trading strategies are more appropriate.

Keywords: price predictability; multifractality; informational efficiency; soybean complex.
JEL classification: G14; Q11; C12.

1. Introduction

Price predictability is central in all theoretical works on market efficiency. Fama (1965) advanced the Efficient Market Hypothesis (EMH) which — in its weak form — suggests that investors are rational and current prices reflect all information about past prices[2]. Therefore, from a statistical viewpoint, prices are random walk processes and (log-) price changes are independent and identically distributed (i.e. white noise processes). For Samuelson (1965), a less restrictive statistical characterization of a price series in an informationally efficient market is that of a martingale process. Under both the random walk and the martingale characterization, however, returns in informationally efficient markets do not possess any statistically significant serial dependence structure and, as such, they are very difficult to forecast even in the short-run (e.g. (Kristoufek, 2018; Ftiti et al., 2021)).

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[2] A market is informationally efficient if it completely processes all information relevant to the fundamental price generation (i.e. a market in which all the available information is fully reflected in prices). Depending on the type of information available, the EMH is distinguished into three forms: weak (information about past prices), semi-strong (public information), and strong (all information, including private one) (Kristoufek, Vosvrda, 2016).

The investor rationality assumption underlying the works of Fama (1965) and Samuelson (1965) has been challenged from researchers in the field of Behavioral Finance. Shefrin (2000) and Shiller (2003), among others, pointed out that human psychology (fear and greed) and heuristic-driven biases influence the behavior of market participants resulting, thus, in market inefficiency. The Adaptive Market Hypothesis (Lo, 2004) accepts that investors are mainly rational. At the same time, however, it emphasizes that market participants often times make mistakes, they learn from them, and they base their predictions on trial and error. An implication of the works of Shefrin (2000), Shiller (2003), and Lo (2004) is that prices may exhibit short- and/or long-range dependence; investors, therefore, may "beat" the market and earn abnormal profits by devising possibly non-linear statistical models for improved price forecasting and derivatives valuation.

The informational efficiency of commodities futures markets is of keen interest for policy makers, practitioners (commodity fund managers, speculators, hedgers), and research economists. Policy makers strive to ensure the long-run sustainable growth of these markets and to create an environment that fosters fair prices and optimal allocation of scare resources. Commodity fund managers study the price dynamics to achieve the desired level of diversification and to exploit any upside potential. The presence and the type of price dependence (e.g. positive or negative) is critical in a speculator's decision to take a short or a long position in a futures market. Hedgers, form expectations about the basis to determine the right time for offsetting a futures contract. The empirical analysis of price series in commodities futures markets allows research economists to test the validity of competing theoretical propositions.

The investigation of price predictability in financial markets has been pursued with a variety of statistical/econometric tools including Rescaled Range Analysis (Hurst, 1951), standard Autocorrelation and Integration tests, Variance Ratio tests (Lo, MacKinlay, 1988), Fractional Integration tests (Robinson, 1994), and Detrended Fluctuation Analysis (DFA) (Peng et al., 1995). All these approaches tend to view serial dependence as a global feature of a stochastic process. Thus, they are suitable only for monofractal time series where the intensity and the pattern of serial dependence do not vary with the magnitude of the underlying fluctuations (small, medium, large). To address this limitation a number of alternative approaches such as the Wavelet Transform Modulus Maxima (WTMM) (Muzy et al., 1993), the Multifractal Detrended Fluctuation Analysis (MFDFA) (Kantelhardt et al., 2002), and the Multifractal Detrended Moving Average (MFDMA) (Gu, Zhou, 2010) have been proposed in the relevant literature.

Over the last 15 years there has been a number of empirical works on multifractality in commodities (energy, agricultural, currency, precious metals, and cryptocurrencies) prices. Bolgorian and Gharli (2011), Wang et al. (2011) focused on gold; He and Chen (2010), Li and Lu (2011) on agricultural commodities; Stošić et al. (2015) on exchange rates; Stošić et al. (2019) on cryptocurrencies; and Gu et al. (2010), Fernandes et al. (2020) and Ftiti et al. (2021) on energy commodities. Also, certain works offered comparisons among commodities markets as well as among commodities and stock markets, e.g. (Al-Yahyaee et al., 2018; Telli, Chen, 2020; Diniz-Maganini, Rasheed, 2022). The empirical results of the earlier works suggest that the multifractal behavior is a common feature of commodities prices; returns may exhibit persistence or anti-persistence (depending on the magnitude of the underlying fluctuations) and, thus, they are predictable.

Against this background the objective of the present work is to investigate price multifractality and informational efficiency in the futures markets of the US soybeans complex. Soybeans ("the King of Beans") is among the few complete protein vegetable-based foods. It is processed into meal and oil and it is found in a very large number of edible and non-edible products ranging from animal feed, cooking oil, vegan food, and soybean milk to biodiesel and other industrial applications. Brazil and the USA are the top producers and exporters while China and Europe are the top importers (OECD-FAO, 2021).

Soybeans, soybean oil, and soybean meal are highly commoditized and are traded as investment tools in a number of exchanges around the globe. The futures markets for soybeans and its products in the US are among the oldest and the most liquid ones. Hedgers and speculators in the soybean complex may trade individual commodities (e.g. soybean meal) or the so-called crush spread (that is, the difference between the combined value of meal and oil and the value of soybeans used to produce them). The crush spread is considered as a gauge of the gross processing margin (GPM) and it drives production decisions (such as capacity utilization and expansion) as well as risk management strategies[3]. Soybean processors typically long the crush spread to establish a floor for their GPM; speculators may long (short) the crush spread depending on whether they expect it to get wider (narrower) in the future.

He and Chen (2010), using MFDFA and daily data over 1993 to 2009, found some evidence of multifractality in the US futures market for soybeans but no evidence of it in that for soybean meal. The pattern of multifractality they reported is consistent with persistence under all small, medium, and large fluctuations. Yin and Wang (2021), using MFDFA and daily data over 2000 to 2019, concluded that the futures prices for soybean in the US exhibit multifractality and (weak) persistence under medium fluctuations. Johnson et al. (1991), Simon (1999), and Mitchell (2010) investigated the informational efficiency of the futures market for the crush spread using standard technical analysis (i.e. returns deviations from the moving average and trading rules regarding trade lengths, entry and exist limits). Johnson et al. (1991) and Simon (1999) found that a large percentage of such trades can be profitable. Mitchell (2010), in contrast, concluded that profitability is very unstable and the market for crush spread is largely efficient.

This work makes three main contributions to the relevant literature.

(a) It analyzes and compares all four markets in the US soybean complex.

(b) It employs a barrage of formal statistical tests to investigate the presence of multifractality and market inefficiency. All earlier works, either on stock or on commodity markets, present a large number of statistics but draw conclusions using visual inspection only. There is no way, therefore, to tell whether the reported departures from monofractality and/or informational efficiency are genuine features of the respective price series or just the outcome of noise in the data.

(c) It assesses the degree of inefficiency through a distance-based measure that utilizes information about the pattern of dependence across the entire distribution of price fluctuations (small, medium, and large). The vast majority of earlier works do not even attempt to measure informational efficiency. A few others, e.g. (Wang et al., 2009; Mensi et al., 2019) rely on an ad hoc index that is likely to result into incorrect (upwards biased) estimates of inefficiency.

In what follows section 2 presents the analytical framework and section 3 the data. Section 4 presents the empirical models and the empirical results, while section 5 offers conclusions suggestions for future research.

[3] See https://www.cmegroup.com/education/files/soybean-crush-reference-guide.pdf.

2. Analytical framework

Let x_t $(t = 1, 2, \ldots, T)$ be a weakly stationary stochastic process (here, a time series of price log-returns). The MFDFA algorithm involves the following five steps[4].

(a) Determine the "profile" (y_t) of x_t as

$$y_k = \sum_{t=1}^{k} (x_t - \bar{x})', \tag{1}$$

where $k = 1, 2, \ldots, T$ and $\bar{x}$ is the average of x_t over the total sample[5].

(b) Divide the "profile" into $T_s = [T/s]$ non-overlapping segments of equal length s. Since T is not necessarily a multiple of s, a number of observations at the end may remain outside of the segments. To avoid discarding that part of the time series, exactly the same procedure is repeated starting from the opposite end. Thereby, the number of segments will double ($2T_s$) and none of the observations will be excluded.

(c) For each segment (v) estimate the local trend by fitting a polynomial of order m ($P_{v,m}(j)$) and compute the local variance as

$$F^2(s,v) = \frac{1}{s} \sum_{j=1}^{s} \left(y_{(v-1)s+j} - P_{v,m}(j) \right)^2 \tag{2a}$$

for $v = 1, 2, \ldots, T_s$, and

$$F^2(s,v) = \frac{1}{s} \sum_{j=1}^{s} \left(y_{T-(v-T_s)s+j} - P_{v,m}(j) \right)^2 \tag{2b}$$

for $v = T_s + 1, T_s + 2, \ldots, 2T_s$.

(d) Average over all $2T_s$ segments to obtain the q order fluctuation function ($F_q(s)$) as

$$F_q(s) = \left(\frac{1}{2T_s} \sum_{v=1}^{2T_s} F^2(s,v)^{q/2} \right)^{1/q} \tag{3a}$$

for any real value $q \neq 0$ and

$$F_0(s) = \exp\left(\frac{1}{4T_s} \sum_{v=1}^{2T_s} \ln F^2(s,v) \right) \tag{3b}$$

for $q = 0$. The parameter q enables one to distinguish between local periods (segments) with small and large fluctuations. In particular, negative (positive) values of q amplify segments with small (large) fluctuations. For $q = 2$, the MFDFA becomes a conventional DFA.

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4 The MFDFA is among the most effective methods for investigating multifractality. At the same time, it is the easiest to implement and the most robust, e.g. (Thompson, Wilson, 2016).

5 Thomson and Wilson (2016) call the price log returns "disaggregated data" and the profile "aggregated data".

(e) Repeat steps (c) and (d) for several time scales (i.e. for different values of s) to determine the scaling behavior of the fluctuation function[6]. An increasing $F_q(s)$ for large values of s in a power-law, $F_q(s) \sim s^{h(q)}$, suggests that y_t is a long-range power law correlated time series. If the scaling exponent $h(q)$ is independent of q (i.e. identical for all segments), y_t is monofractal; if it is q-dependent, y_t is multifractal. In the latter case, $h(q)$ is called generalized Hurst exponent and it reflects differences in the scaling behavior between segments with small $(q<0)$ and with large $(q>0)$ fluctuations. For a process with stationary increments such as y_t, $h(2)$ equals the standard Hurst exponent (H) that reflects long-range (global) dependence.

$h(q)$ can be estimated as the slope of the log-log regression of $F_q(s)$ on s. When $h(q)=0.5$ the time series is, locally, a standard Brownian motion. When $0<h(q)<0.5$, the process exhibits anti-persistence; that means, locally, a positive increment in the process is more likely to be followed by a negative increment in the next non-overlapping time interval; this tendency for mean reversion results in sample paths with a rough structure. Anti-persistence is consistent with overreaction to incoming information (Fernadez, 2010). When $0.5<h(q)<1$, the process exhibits persistence; that means, locally, successive non-overlapping increments are likely to have the same sign; this tendency results in sample paths with a smooth structure. Note that $h(q)$ is a monotonic (decreasing) function of q (Ihlen, 2012).

An alternative way to investigate the presence and the pattern of multifractality is through the singularity spectrum $f(a)$ that summarizes the temporal variation of the generalized Hurst exponents (that is, the singularity content of the time series). The singularity spectrum can be obtained through the Legendre transform

$$\alpha = \frac{\mathrm{d}\tau(q)}{\mathrm{d}q} = \frac{\mathrm{d}(h(q)q-1)}{\mathrm{d}q} = \frac{\mathrm{d}h(q)}{\mathrm{d}q}q + h(q) \tag{4a}$$

and

$$f(\alpha) = q\alpha - \tau(q). \tag{4b}$$

In (4a), $\tau(\mathrm{q})$ is the mass (Renyi) exponent and α is the Holder exponent that characterizes singularities in y_t. For monofractal time series, $\alpha = H$ and $f(a)=1$ for every α (i.e. the singularity spectrum collapses to a single point). Multifractal series, however, yield an $f(a)$ which is a humped function (Ihlen, 2012; Stošić et al., 2019).

Three statistics of the singularity spectrum provide information about the complexity of a time series; namely, its central tendency (mode), its width, and its shape (Shimizu et al., 2002). The central tendency (α_0), is the value of the Holder exponent that maximizes $f(a)$ (i.e. the value at which $f(a)=1$). Low (<0.5) values of α_0 indicate that the process tends, globally, to exhibit persistence; it has no “fine structure” and it becomes more regular in appearance. High (>0.5) values indicate that the process tends, globally, to exhibit anti-persistence; it has a “fine structure” reflected in a high degree of roughness (Telli, Chen, 2020; Stošić et al., 2019)[7]. The width $W = \alpha_{\max} - \alpha_{\min}$,

[6] Small (large) values of s correspond to small (large) sub-samples and small (large) sub-samples correspond to small (large) time scales.

[7] Note that for self-affine stochastic processes (i.e. those for which their power spectral density scales as power of their frequency), the standard Hurst exponent (H) is negatively related to roughness (Thompson, Wilson, 2016; Kristoufek, 2018). This, in turn, implies that high values of $h(q)$ tend to be associated with low values of α. Therefore, a value of $a_0 <(>)\ 0.5$ is consistent with a value of $H >(<)\ 0.5$.

captures the degree of multifractality (i.e. the range of possible scaling exponents). High values of W indicate a series with a "rich" structure (i.e. one that involves more fluctuation and risk). The shape asymmetry statistic $A = \frac{\alpha_{\max} - \alpha_0}{\alpha_0 - \alpha_{\min}}$, provides information about the relative importance of small and large fluctuations. A left-skewed singularity spectrum $(A < 1)$ implies that the time series has a multifractal structure that is insensitive to local fluctuations with small magnitudes (i.e. dominance of large fluctuations); a right-skewed one $(A > 1)$ indicates dominance of small fluctuations. A stochastic process with a high value of α_0, a wide range of scaling exponents, and a right-skewed spectrum is more "complex" (less predictable) than one with the opposite characteristics.

For an informationally efficient market it is the case that $h(q) = H = 0.5,\ \forall q \in [q_{\min}, q_{\max}]$. Based on this, Wang et al. (2009), Al-Yahyaee et al. (2018), and Mensi et al. (2019) suggest that the degree of informational inefficiency can be captured through the market deficiency measure

$$MDM = 0.5\big[(h(q_{\max}) - 0.5) + (0.5 - h(q_{\min}))\big] = 0.5\big[h(q_{\max}) - h(q_{\min})\big] = 0.5 \cdot \Delta h.$$

The *MDM* relies exclusively on information on the values of the generalized Hurst exponents under extreme large and extreme small fluctuations. As such, it fails to take into account a market's informational efficiency under milder shocks (which in any case are much more common). Given that $h(q)$ is monotonically decreasing in q, the *MDM* is likely to result into a biased (artificially inflated) estimate of market inefficiency. More importantly, a price series can be monofractal (i.e. with degree of multifractality $\Delta h = 0$) and, at the same time, the relevant market can be informationally inefficient[8]. To address this problem, and following Kristoufek (2018), the present work relies on a Modified MDM (denoted as *MMDM*) that utilizes information across all values of q_s. The *MMDM* is defined as

$$MMDM = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{h(q(i)) - 0.5}{R}\right)^2}, \tag{5}$$

$$q(i) = q(\min), \ldots, q(\max).$$

In (5), 0.5 is the expected value of the generalized Hurst exponent at fluctuation level $q(\mathrm{i})$ in the ideal state (informational efficiency); R is the theoretical range of $h(q)$ values (for a weakly stationary time series $R = 1$); n is the number of fluctuation levels considered. The measure receives values in $[0, 0.5]$. The value 0 corresponds to an informationally efficient market (i.e. where $h(q(i)) = 0.5\ \ \forall q(i) = q(\min), \ldots, q(\max)$). The value 0.5 corresponds to a completely inefficient one (i.e. where $h(q(i)) = 1\ \ \forall q(i) = q(\min), \ldots, q(\max)$ (perfect persistence) or where $h(q(i)) = 0\ \ \forall q(i) = q(\min), \ldots, q(\max)$ (perfect anti-persistence) or where $h(q(i)) = 1$ for some i and $h(q(i)) = 0$ for all the rest[9]).

[8] Multifractality is sufficient, not necessary condition for dependence. For example, the monofractal time series with $h(q) = H = 0.3\ \forall q$ exhibits global anti-persistence.

[9] Observe that under informational inefficiency and $R = 1$ it is the case that $MMDM = \sqrt{n^{-1}\sum_{i=1}^{n}(0.5/R)^2} = 0.5$ (the upper bound of the measure).

3. The data

The data for the empirical analysis are closing daily prices of soybeans (in $ per bushel), soybean meal (in $ per short ton), and soybean oil (in $ per pound). They have been obtained from Yahoo Finance and they refer to the period 1/1/2015 to 31/12/2021. Crushing a bushel (60 pounds) of soybean typically yields 44 pounds of 48 percent protein meal and 11 pounds of oil. Given that a soybean futures contract is 5000 bushels, a meal contract is 100 short tons, and an oil contract is 60000 pounds, a combination that approximates more accurately the yields involves 11 meal, 9 oil, and 10 soybean contracts[10]. Accordingly, the crush spread in $ per bushel of soybeans processed is

$$\frac{\begin{bmatrix}(\text{meal price, in \$/ton})\times 0.022\times 11+(\text{oil price, in \$/pound})\times \\ \times 11\times 9-(\text{soybean price, in \$/bushel})\times 10\end{bmatrix}}{10}. \tag{6}$$

The main price drivers in the soybean complex are: (a) the supply in USA and Brazil; (b) the prices of grains; (c) the demand for livestock (soybean meal is a favored feedstock for pigs); (d) the conditions in the wider energy market; and (e) the price for palm oil (the main substitute for soybean oil). Figure 1 presents the evolution of soybeans, meal, and oil prices as well as that of the crush spread. The prices of soybeans and its products follow similar trends (although, over short intervals of time, they may diverge from each other). The main reason behind the spike from early 2020 to mid-2021 was a production shortfall due to poor weather in South America[11]. Since soybeans, meal and oil are intrinsically interlinked commodities their prices are expected to move in the same direction. Because of this, investors perceive trading the crush spread as less risky relative to holding outright futures positions. This perception is reflected in the initial margin requirements (they are lower for trading the spread than individual commodities futures contracts). Nevertheless, when the three prices in the complex diverge the crush spread may become very volatile. Here, the crush spread increased from about 0.5 $ per bushel in early 2016 to about 2.6 $ per bushel in mid-2018 to fall to about 1.6 $ per bushel in the most recent months.

The four futures prices series have been used to compute price log-returns as $r_{it} = \ln(p_{it} / p_{i,t-1})$, where i = soybeans, soybean meal, soybean oil, and the crush spread. Table A1 in the Appendix presents summary statistics and tests on the distribution of price log-returns. Soybeans and oil exhibit negative and statistically significant skewness pointing to the presence of a few very large negative shocks. Soybean meal exhibits positive skewness while the empirical distribution of the crush spread is symmetric. All four series show excess kurtosis pointing to the presence of a broad distribution (i.e. one with fat tails). The excess kurtosis is most notable for the crush spread[12]. The null of normality is strongly rejected in all cases. Table A2 in the Appendix presents

[10] See https://www.cmegroup.com/trading/agricultural/grain-and-oilseed/soybean-crush-spreads.html.

[11] See https://openknowledge.worldbank.org/bitstream/handle/10986/36350/CMO-October-2021.pdf.

[12] The simple Pearson's correlation coefficients between the price log-returns pairs (soybeans, meal), (soybeans, oil), and (oil, meal) are 0.77, 0.56, and 0.14, respectively, suggesting that co-movement, although positive, is far from perfect. This, in turn, explains the much higher volatility and degree of excess kurtosis in the crush spread relative to the other three time series.

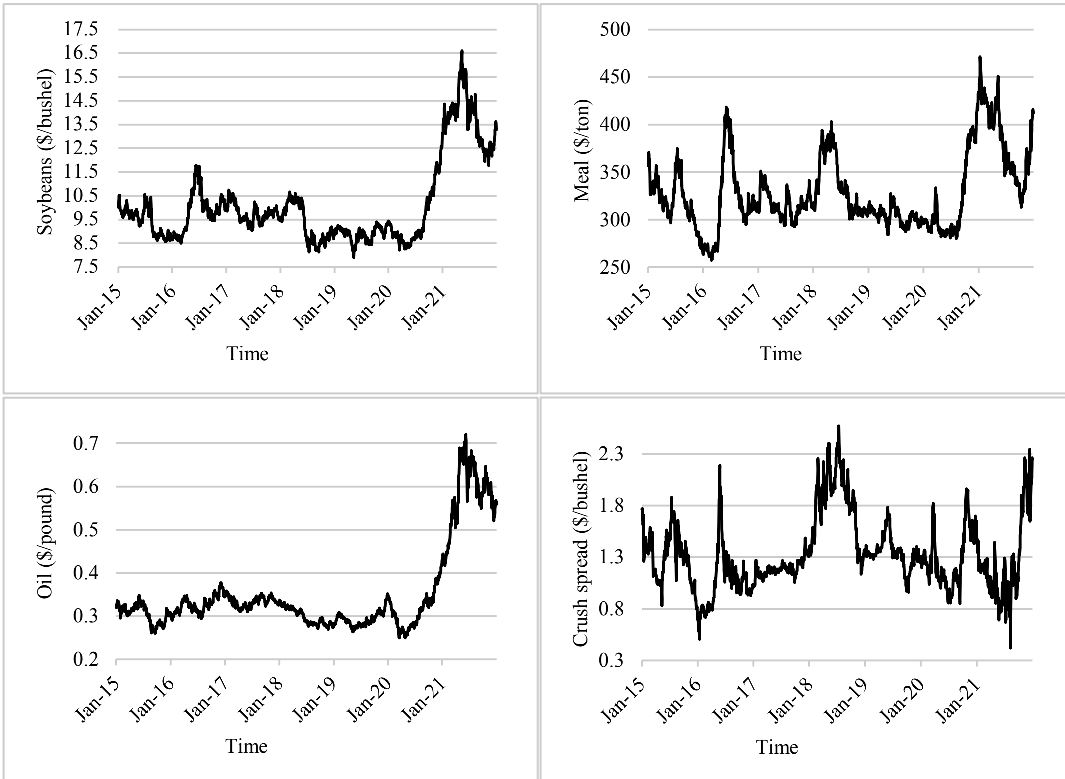

Fig. 1. Soybeans, meal, and oil prices, and the crush spread over 2015–2021

unit root tests on the price log-returns. The null hypothesis of level stationarity is not rejected at any reasonable level of significance.

4. The empirical models and the empirical results

For the empirical implementation of the MFDFA, the time scales (s) have been set into [10, T/4] as in (Kantelhardt et al., 2002; Da Silva Filho et al., 2018); the range of q to $[-10, 10]$ as in (Wang et al., 2011; Milos et al., 2020); and the polynomial order to $m = 1$ as in (Li, Lu, 2011; Milos et al., 2020)[13]. The individual and joint hypotheses tests have been conducted using a Wald-type statistic

$$\Omega = (\Pi \hat{C})'(\Pi \hat{V}_C \Pi')^{-1}(\Pi \hat{C}), \tag{7}$$

where Π is the restrictions' vector, C is the parameters' vector, and $\hat{V}_C$ is the bootstrap estimate of their variance-covariance matrix (Patton, 2013). Under a null, Ω follows the χ^2-distribution with degrees of freedom equal to the number of restrictions.

The MFDFA has been carried out using package MFDFA (Laib et al., 2019) in *R*. Table 1 (panel (a)) presents the generalized Hurst exponents at different qs for each of the four time

[13] Experimentation with different ranges of s and q and different values of m (2 and 3, in particular) has not led to any notable qualitative change in the results.

series along with information on whether these exponents are individually different from 0.5 at the conventional levels of significance (i.e. each individual null hypothesis is $h(q(i)) = 0.5$ for $q(i) = -10,...,10$). Table 1, in addition, shows the values of Δh for each of the four series along with information on whether these are individually different from 0 at the conventional levels of significance (i.e. each individual null hypothesis is $\Delta h = 0$). The statistical inference in Tables 1–4 below for both the individual and the joint tests relied on block-bootstrap (Politis, Romano, 1994) with 1000 replications.

Table 1. Generalized Hurst exponents

q	Original series (a)				Shuffled series (b)				Surrogated series (c)			
	Soybeans	Meal	Oil	Crush spread	Soybeans	Meal	Oil	Crush spread	Soybeans	Meal	Oil	Crush spread
–10	0.60*	0.61**	0.57	0.78***	0.54	0.55	0.66	0.57	0.63	0.57	0.73	0.56
–9	0.59	0.60*	0.56	0.77***	0.53	0.54	0.65	0.57	0.62	0.56	0.73	0.55
–8	0.59	0.60*	0.56	0.76***	0.52	0.53	0.64	0.56	0.62	0.56	0.72	0.54
–7	0.58	0.59*	0.55	0.74***	0.51	0.52	0.63	0.55	0.62	0.56	0.71	0.54
–6	0.57	0.58	0.54	0.73***	0.50	0.52	0.63	0.53	0.62	0.55	0.71	0.53
–5	0.56	0.58	0.53	0.70***	0.49	0.51	0.61	0.52	0.62	0.55	0.70	0.52
–4	0.56	0.57	0.52	0.67***	0.48	0.50	0.60	0.50	0.62	0.55	0.69	0.52
–3	0.55	0.57	0.51	0.63***	0.47	0.49	0.59	0.49	0.61	0.55	0.69	0.51
–2	0.54	0.57	0.51	0.59*	0.47	0.48	0.58	0.47	0.61	0.56	0.68	0.50
–1	0.54	0.57	0.51	0.54	0.47	0.48	0.57	0.46	0.61	0.56	0.68	0.49
0	0.54	0.58*	0.51	0.49	0.47	0.48	0.56	0.44	0.61	0.56	0.67	0.49
1	0.54	0.58*	0.51	0.44	0.48	0.48	0.55	0.43	0.61	0.56	0.67	0.48
2	0.54	0.58	0.51	0.40**	0.48	0.48	0.54	0.41	0.61	0.56	0.66	0.47
3	0.53	0.57	0.50	0.35***	0.49	0.48	0.53	0.40	0.61	0.57	0.65	0.46
4	0.52	0.55	0.49	0.32***	0.50	0.48	0.53	0.38	0.60	0.57	0.64	0.45
5	0.51	0.54	0.47	0.28***	0.50	0.48	0.52	0.37	0.60	0.57	0.63	0.44
6	0.49	0.53	0.45	0.26***	0.50	0.48	0.52	0.36	0.59	0.57	0.62	0.43
7	0.48	0.52	0.44	0.23***	0.50	0.47	0.52	0.35	0.59	0.57	0.61	0.43
8	0.47	0.51	0.42	0.22***	0.50	0.47	0.51	0.34	0.58	0.57	0.60	0.42
9	0.47	0.51	0.41	0.20***	0.50	0.47	0.51	0.33	0.58	0.57	0.59	0.41
10	0.46	0.50	0.40	0.19***	0.49	0.46	0.50	0.33	0.57	0.57	0.58	0.41
Δh	0.14*	0.11	0.17**	0.58***	0.04	0.08	0.15	0.25	0.06	0.00	0.15	0.15
	Joint tests											
$h(-10) = \ldots = h(-1) = 0.5$	75.03***	90.62***	74.02***	121.71***								
$h(1) = \ldots = h(10) = 0.5$	71.57***	69.84***	101.9***	98.23***								

Notes. (a) *, **, and *** denote statistical significance at the 10, the 5, and the 1 percent level, respectively. The individual null hypotheses are $h(q_i) = 0.5$ for $i = -10,...,10$ and $\Delta h = 0$.
(b) The part joint tests show the respective χ^2-statistics. The critical values of the χ^2-distribution with 9 degrees of freedom are 14.604, 16.991, and 21.666 at the 10, the 5, and the 1 percent level, respectively.

The null hypothesis $h(2)=0.5$ is not rejected at the conventional levels of significance for soybeans, meal, and oil suggesting that none of these three time series exhibits a tendency for global persistence or anti-persistence (in other words, none of them exhibits long-range dependence/memory). The same hypothesis, however, is rejected (marginally at the 5 percent level) for the crush spread. Given that the empirical value of $h(2)$ for the latter is less than 0.5, there is some evidence that the crush spread series exhibits global anti-persistence (i.e. the crush spread is, globally, a pink noise process). In two out of four cases (oil and the crush spread) Δh is statistically significant at the 5 percent level offering, thus, strong evidence in favor of multifractality. For soybeans, Δh is statistically significant at the 10 percent level, while for meal Δh is not statistically significant.

As expected, the generalized Hurst exponents tend to decrease with the magnitude of fluctuations. For soybeans, oil, and the crush spread and for $q>(<0)$ the generalized Hurst exponents tend to be lower (higher) than 0.5. For meal, the generalized Hurst exponents are all higher than 0.5. For the crush spread, the large majority of the differences between the $h(q)$ values and 0.5 are statistically significant at the 1 percent level. For soybeans, meal and oil (and for extreme fluctuations only), less than a handful of these differences are statistically significant even at the 10 percent level. Barring the crush spread, the statistical evidence obtained from the individual tests about local persistence or anti-persistence is, to a very large extend, inconclusive. To shed more light on this important issue, the paper employs joint tests. In particular, given that high (above 0.5) values of $h(q)$ correspond to low values of q, it tests the following two joint hypotheses

$$h(-10)=...=h(-1)=0.5\,, \tag{8a}$$

$$h(1)=...=h(10)=0.5\,. \tag{8b}$$

For all series, (8a) and (8b) are strongly rejected. One may, therefore, conclude that the prices of soybeans, oil, and of the crush spread exhibit local persistence (anti-persistence) for low (high) values of q. The returns of the meal price, however, are likely to show local persistence (black noise) for all large and small fluctuations.

Kanterlhardt et al. (2002) highlighted two major sources of multifractality, namely, the differences in serial dependence under small and large fluctuations and the non-Gaussian probability distribution of increments (i.e. the presence of fat-tails). The relative contribution of each source can be investigated using random shuffling and phase-randomization. The random shuffling puts the data from the original series into a random order to ensure that all temporal dependence is destroyed while the probability density function remains exactly the same. The phase-randomization weakens non-Gaussianity but preserves the linear properties of the original data. If the degree of multifractality (Δh) in the shuffled series is significantly smaller than that in the original series whereas it decreases only slightly with phase-randomization, one may conclude that multifractality is caused by differences in serial dependence under small and large fluctuations. If the degree of multifractality in the surrogated series (the one resulting from phase-randomization) is significantly smaller than that in the original series whereas it decreases only slightly with shuffling, one may conclude that multifractality is caused by a fat-tailed distribution. Finally, if Δh decreases significantly after both shuffling and phase-randomization one may conclude that differences in correlations as well as fat-tails contribute to multifractality.

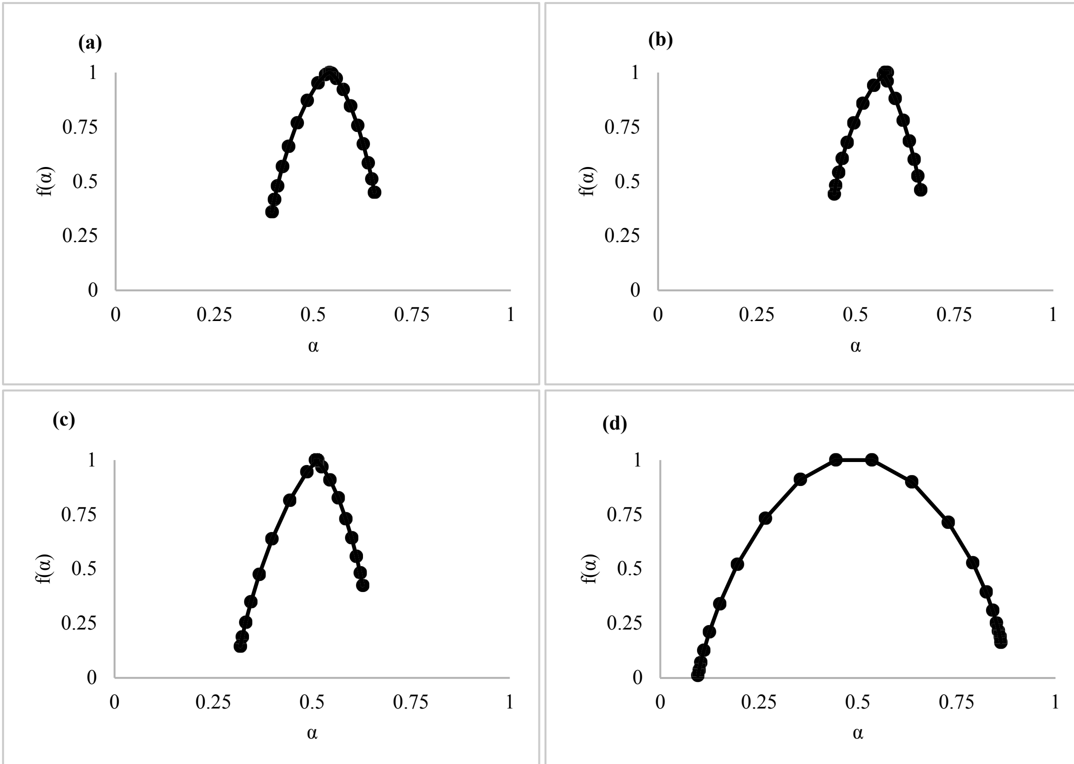

Fig. 2. Singularity spectra: (a) soybeans; (b) meal; (c) oil; (d) crush spread

Table 1 (panels (b) and (c)) shows the generalized Hurst exponents for the shuffled and the surrogated series, respectively[14]. In all cases, random shuffling and phase-randomization have reduced the degree of multifractality. Judging from the relative magnitude of reductions, however, it appears that for meal and the crush spread the contribution of fat tails to multifractality is more important than the differences in serial dependence under small and large fluctuations. The opposite appears to hold for soybeans.

Figure 2 (panels (a), (b) (c), and (d)) presents the singularity spectra for soybeans, meal, oil, and the crush spread, respectively. Table 2 shows the results from individual and joint tests on the three complexity parameters. The mode values (a_0) lie in $[0.49, 0.58]$. Three (those for soybeans, oil, and the crush spread) are not different from 0.5 at any reasonable level of significance; for meal, the difference is significant at the 10 percent level. The information from the mode values is, therefore, largely consistent with the evidence from Table 1 (panel (a)). The evidence from the individual tests is further reinforced from the results of the joint test (the null hypothesis all four mode values are equal to each other is not rejected at the conventional levels of significance). Note that central tendency measures ranging from 0.5 to 0.6 have been reported by Stošić et al. (2019) for cryptocurrencies, Fernandes et al. (2020) and Ftiti et al. (2021) for energy commodities, and Yin and Wang (2021) for soybeans. The spectrum width values (W) lie in $[0.22, 0.77]$

14 The surrogated series were obtained through the package *nonlinearTseries* (Garcia, Sawitzki, 2021) in R while the shuffled with the R function *sample* (Becker et al., 1988) of the base package (https://www.R-project.org/).

and they are different from zero on the basis of the individual tests. The joint null that W is the same for all markets is strongly rejected; the joint null, however, that W is the same only for soybeans, meal, and oil is not rejected. The test results on W, therefore, are perfectly consistent with what has been already transpired from Table 1 (panel (a)), i.e. all four series are multifractal and the degree of multifractality for the crush spread in higher relative to the rest. The values of the asymmetry statistic (A) lie in $[0.61, 0.94]$. None of them is statistically different from 1 and the joint null that A is the same across all four series is not rejected at the conventional levels suggesting that small and large fluctuation are equally important for the evolution of prices in the soybean complex. From Table 2 follows that, first, the soybean, the meal, and the oil markets are equally complex and, second, the crush spread market is more complex relative to the rest only because it involves a higher level of risk.

Table 2. Complexity parameters

	Soybeans (1)	Meal (2)	Oil (3)	Crush spread (4)	Joint tests	
a_0	0.54	0.58^{*}	0.51	0.49	(1) = (2) = (3) = (4)	2.20
W	0.26^{***}	0.22^{**}	0.31^{***}	0.77^{***}	(1) = (2) = (3) = (4)	16.34^{***}
					(1) = (2) = (3)	0.48
A	0.78	0.66	0.61	0.94	(1) = (2) = (3) = (4)	0.02

Notes. (a) *, **, and *** denote statistical significance at the 10, the 5, and the 1 percent level, respectively. The individual null hypotheses are $a_0 = 0.5$, $W = 0$, $A = 1$.
(b) The part joint tests show the respective χ^2-statistics. The critical values of the χ^2-distribution with 3 degrees of freedom are 6.251, 7.815, and 11.345 at the 10, the 5, and the 1 percent level, respectively. The critical values of the χ^2-distribution with 2 degrees of freedom are 4.605, 5.991, and 9.210 at the 10, the 5, and the 1 percent level, respectively.

Table 3 shows the values of the *MMDM*s over the total sample. They lie in $h(q)$. On the basis of the levels of statistical significance, the individual null of informational efficiency is strongly rejected for meal and the crush spread and weakly rejected for soybeans and oil. The joint null that the degrees of informational inefficiency are the same across all four markets is also strongly rejected. The null, however, that the *MMDM*s are the same for soybeans, meal, and oil is not. One may conclude, therefore, that — for the period 2015 to 2021— no market has been informationally efficient and that the crush spread one has been more inefficient relative to the rest.

Table 4 shows the values of the *MMDMs* by year. The crush spread market tends to have higher *MMDM* values and it is the one where that null of informational efficiency is more often rejected at the conventional levels. The joint null, however, that the degree of informational efficiency is the same across all four markets or it is the same across the soybeans, the meal, and the oil ones is rejected only for the year 2020.

5. Conclusions

The objective of the present work has been to assess informational efficiency in the futures markets of the US soybeans complex. This has been pursued using daily prices over 2015 to 2021 and Multifractal Detrended Fluctuation Analysis, an econometric tool that allows the intensity and the pattern of serial dependence locally to vary with the magnitude underlying fluctuations.

Table 3. The modified market deficiency measure (total sample)

	Soybeans (1)	Meal (2)	Oil (3)	Crush spread (4)	Joint tests	
MMDM	0.05^{*}	0.07^{***}	0.05^{*}	0.21^{***}	(1) = (2) = (3) = (4)	15.78^{***}
					(1) = (2) = (3)	0.39

*Notes***.** (a) *, **, and *** denote statistical significance at the 10, the 5, and the 1 percent level, respectively. Each individual null hypothesis is $MMDM = 0$.
(b) The part joint tests show the respective χ^2-statistics. The critical values of the χ^2-distribution with 3 degrees of freedom are 6.251, 7.815, and 11.345 at the 10, the 5, and the 1 percent level, respectively. The critical values of the χ^2-distribution with 2 degrees of freedom are 4.605, 5.991, and 9.210 at the 10, the 5, and the 1 percent level, respectively.

Table 4. The modified market deficiency measure (by year)

MMDM	Soybeans (1)	Meal (2)	Oil (3)	Crush spread (4)	Joint tests	
2015	0.17^{***}	0.23^{***}	0.06	0.28^{***}	(1) = (2) = (3) = (4)	4.67
					(1) = (2) = (3)	3.21
2016	0.16^{**}	0.06	0.14^{**}	0.28^{***}	(1) = (2) = (3) = (4)	4.40
					(1) = (2) = (3)	1.76
2017	0.10	0.15^{**}	0.18^{***}	0.17^{*}	(1) = (2) = (3) = (4)	1.05
					(1) = (2) = (3)	1.00
2018	0.08	0.15^{**}	0.06	0.24^{***}	(1) = (2) = (3) = (4)	3.01
					(1) = (2) = (3)	1.14
2019	0.14^{**}	0.08	0.15^{**}	0.08	(1) = (2) = (3) = (4)	0.76
					(1) = (2) = (3)	0.61
2020	0.22^{***}	0.36^{***}	0.12^{*}	0.27^{***}	(1) = (2) = (3) = (4)	6.48^{*}
					(1) = (2) = (3)	6.34^{**}
2021	0.05	0.16^{**}	0.12^{*}	0.24^{***}	(1) = (2) = (3) = (4)	3.19
					(1) = (2) = (3)	1.86

*Notes***.** See Notes to the Table 3.

The empirical results suggest:

(a) The input (soybeans) and the output (soybean meal and oil) prices are not likely to exhibit long-range dependence either on the basis of the Hurst exponent or on the basis of the mode of the singularity spectrum. There is, however, (weak) evidence of global anti-persistence for the crush spread series on the basis of the Hurst exponent only.

(b) The prices of soybeans, soybean oil and of the crush spread are multifractal either on the basis of the degree of multifractality or on the basis of the width of the singularity spectrum. The price of soybean meal is multifractal on the basis of the width of the singularity spectrum only. As far as the pattern of multifractality is concerned, soybeans, soybean oil, and the crush spread prices exhibit local anti-persistence (persistence) under large (small) fluctuations while that soybean meal is likely to show local persistence for all large and small fluctuations. In this respect, the findings here are not in agreement with those of He, Chen (2010) and Yin, Wang (2021) according to which the futures price of soybeans in the US exhibit local persistence.

As noted by Kristoufek and Vosvrda (2013, 2014), there is no inconsistency whatsoever between the absence of global dependence and the presence of local persistence or anti-persistence.

Such finding is simply an indication that short-term trends such as "bear" (i.e. a declining market with a negative mood) and "bull" (i.e. an increasing market with a positive mood) are not necessarily reflected in the global characteristics of a price series; long-range dependence is likely to vanish through the interaction of supply and demand (arbitrage).

The pattern of local serial dependence is an important piece of information for traders. In soybean meal market (where prices exhibit persistence for small and large fluctuations) trend-oriented trading strategies such as those based on moving averages are likely to be profitable. In the markets of soybeans, soybean oil, and of the crush spread (where prices show persistence for small fluctuations and anti-persistence for large ones), however, it appears that traders should condition their strategies on the magnitude of the underlying shocks; specifically, they should use trend-oriented (oscillatory) strategies for small (large) fluctuations.

(c) None of the four markets considered has been informationally efficient. The most inefficient among them has turned out to be that of the crush spread. This observation applies to the results from the whole sample as well as to those from the individual years. The inefficiency of the crush spread market is directly related to the level of price risk (reflected in a higher degree of multifractality and in a much wider singularity spectrum relative to the other three markets). This finding is surprising because (as already noted in Section 3) investors perceive trading of the crush spread as less risky relative to holding outright futures positions. Nevertheless, the higher volatility of the crush spread series, over 2015–2021, has been already evident from Figure 1 and from the descriptive statistics and the tests on price log-returns shown in Table A1 (Appendix). Whether the lower margin can compensate for the risk involved in the crush spread market is an empirical question. In any case, the "rich" structure of the crush spread series points to ample opportunities for speculation in the crush spread market something that appears to be consistent with results of technical analysis obtained earlier by Johnson et al. (1991) and Simon (1999).

Future research may assess and compare informational efficiency in other sets intrinsically interlinked markets. Examples are the futures markets of crude oil and its products, and futures markets of electricity and natural gas or of electricity and coal. The gross operational margins for these complexes are the crack, the spark, and the dark spread, respectively.

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Received 10.01.2022; accepted 15.04.2022.

Appendix

Table A1. Descriptive statistics and tests on the distributions of price log-returns

Statistic	Soybeans	Meal	Oil	Crush spread
Mean	0.000	0.000	0.000	0.000
Median	0.000	–0.001	0.000	0.001
SD	0.012	0.014	0.014	0.055
Minimum	–0.086	–0.083	–0.093	–0.730
Maximum	0.064	0.075	0.066	0.688
1st quartile	–0.007	–0.008	–0.008	–0.021
3rd quartile	0.007	0.007	0.009	0.021
Skewness	–0.246***	0.126**	–0.155***	0.080
Kurtosis	6.924***	5.985***	5.099***	44.983***
Normality	0.961***	0.966***	0.983***	0.753***

*Notes***.** **, and *** denote statistical significance at the 5, and the 1 percent level, respectively. Skewness, kurtosis, and normality been evaluated the tests by D'Agostino (1970), Anscombe, Glynn (1983), and Shapiro, Wilk (1965), respectively.

Table A2. Unit root tests on price log-returns

	Soybeans	Meal	Oil	Crush spread
With constant	0.165	0.112	0.259	0.071

Notes. The critical values for the KPSS test (Kwiatkowski et al., 1992) with a constant are 0.347, 0.436, and 0.739 at the 10, the 5, and the 1 percent level, respectively.