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Energy commodities: A study on model selection for estimating Value-at-Risk

Changes in commodity prices can be transmitted directly to the real economy through changes in the marginal cost of production. Therefore, it is extremely important to create some mechanism to protect against these movements in the commodities futures market. Exposure in this market comes along with tail risk, which must be measured and controlled using a risk measure. To help economic agents, this research provides a common statistical specification that can be used to reliably predict the Value-at-Risk of four important energy commodities. For this, the predictions of a range of 48 competing models, composed of four heteroskedastic specifications, six conditional distributions, and a Markov chain with up to two regimes, were compared using various statistical tests, and the model with the best average results was preferred.

Keywords: commodities; Value-at-Risk; GARCH; Markov-switching; probability distributions.
JEL classification: C46; C53; G13; G32; P18.

1. Introduction

Commodities are used as basic raw materials in the production of many goods for the economy. For this reason, Garratt, Petrella (2021) and Lin et al. (2021) warn that commodity price changes can be transmitted directly to the real economy through changes in the marginal cost of production and consequently cause changes in the aggregate price level (see, also, (Chen et al., 2020)). These price changes can impact interest rates (Çepni et al., 2021; Coletti et al., 2021), exchange rates (Albulescu, Ajmi, 2021, Wang et al., 2022), and also the economic growth of countries (Boateng et al., 2022; Herrera et al., 2019; Liaqat et al., 2022; Mohaddes, Pesaran, 2017; Wang, 2022), although differently, as Liu, Serletis (2021) point out. Among all the inputs, Çepni et al. (2021) and Gong et al. (2022) reveal that oil is one of the main determinants of economic aggregates and deserves to be highlighted. This makes sense, since increases in oil prices can cause increases in the production costs of goods that use large amounts of energy or are directly derived from it (Lin et al., 2021). As a consequence, pressures in other relevant sectors of the economy can be observed (Coletti et al., 2021). For example, Chowdhury et al. (2021) point out that increases in oil prices can cause food prices to rise. Hanson et al. (1993) link this movement to the impact that the price of oil has on the prices of fertilizers, chemicals, transportation,

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and other inputs. While Chen et al. (2010) believe that the increase in food prices can be explained by the increased demand for corn and soybeans for biofuel production as substitutes for oil. Generalizing these facts, Le et al. (2020) show that oil price fluctuations have a significant impact on the prices of other energy commodities, thus evidencing a dependency relationship between their prices. This line of reasoning indicates that there may be a direct contagion effect from oil prices to other commodities (Chowdhury et al., 2021) and this has an impact on the entire industrial chain, affecting the macroeconomy as a whole (Lin et al., 2021).

Since oil price fluctuations can have undesirable consequences for the economy, it is of utmost importance that economic agents set up some sort of protection mechanism against significant variations in their prices. By doing so, for example, companies dependent on this commodity, either directly or indirectly, can mitigate market risks and gain competitive advantages over their competitors. This observation is also valid for individuals who include oil and its derivatives in their consumption baskets (Coletti et al., 2021; Devaguptapu, Dash, 2021) and want to maximize their utility functions. In addition, companies, farmers, investors, investment funds, and the like can also benefit from these protection mechanisms to reduce the market risk of their conventional financial positions (see, as an example, (Ben-Salha, Mokni, 2022; Enwereuzoh et al., 2021; Ji et al., 2020a, b; Kuang, 2022; Liu, Guo, 2022), since commodity prices tend to behave inversely to economic activity. In all these examples, exposure in the commodities futures market becomes advisable to build a protection mechanism, but it should be done with care. Derivatives prices are very volatile (Garratt, Petrella, 2021) and having an exposure in this market involves important risks that should be considered by economic agents. One of them is the risk of extreme events occurring that can cause high financial losses, known as tail risk. Controlling this risk is essential within active risk management. One way to accomplish this task is to use risk measures that are based on the quantiles of a distribution, such as Value-at-Risk (VaR) (Bello et al., 2020). VaR is a powerful tool, but to calculate it accurately requires some effort. In its parametric form, the essence of VaR measurement involves knowing the volatility of the financial asset and the statistical probability distribution of its data. Although it seems simple, having this information is not an easy task, since there are many different models that can be used to estimate volatility and there are many alternative probability distributions available.

Over the past few years, the academic literature has been trying to help economic agents make these choices by providing empirical references that can improve the measurement of VaR. It is quite common to find studies on this topic in assets traded on stock exchanges, see, for example, (Alemohammad et al., 2020; Ardia et al., 2018; Basatini, Rezakhah, 2020; BenSaïda et al., 2018; Mwamba, Mwambi, 2021; Paolella et al., 2019; Sampid et al,. 2018; Segnon, Trede, 2018; Wang et al., 2021; Zolfaghari, Sahabi, 2017). It is also common in cryptocurrencies and exchange rates, see (Ardia et al., 2018, 2019; Caporale, Zekokh, 2019; Chkili, 2021; Maciel, 2021; Segnon, Bekiros, 2020; Siu, 2021; Tan et al., 2021; Wu et al., 2020). But for commodities, studies on this topic are not common, although they do exist. For precious metals, see (Owusu Junior et al., 2022). For oil exclusively, see (Lyu et al., 2017; Salisu et al., 2022). For commodities in general, see (Amaro et al., 2022; Herrera et al., 2017; Laporta et al., 2018). All these works seek to point out ways for economic agents to make the most coherent decision on how to measure the VaR of their financial assets.

This study follows that same path, but with some important innovations to the academic literature, namely:

(a) *Database*. In addition to energy commodities, this research investigates agricultural commodities that can be used to produce biodiesel, which is considered one of its substitute goods.

Thus, this paper covers commodities that are little mentioned in the literature, but that are essential for good risk management. This contribution is important because it seeks to offer previously insufficiently studied alternatives for economic agents to carry out their financial operations or to better understand the behavior of the energy market.

(b) *Common statistical specification* for both commodities. To the best of our knowledge, this is the first work that seeks to provide a common statistical specification that can be reliably used for VaR forecasting for all the commodities studied. For this, a performance ranking is created for all competing specifications by means of the measured prediction accuracy statistics, and the best rated one is chosen. The central idea is to find the statistical specification that has the best average forecast quality for all commodities. This contribution is important because it simplifies the work of economic agents who have computational limitations to perform complex calculations.

(c) *Modeling and estimation*. This research investigates the combined use of different heteroskedastic models with different probability density functions, employing both single and multiple regimes, in estimating the conditional volatility used in VaR forecasting. Here, the contribution covers three fundamental pillars:

(i) whether heteroscedastic models with more complex mathematical structures are more efficient than models with simpler structures;

(ii) whether the use of Markov chains improves the estimation results. Ardia et al. (2018, 2019), Caporale, Zekokh (2019) and Segnon et al. (2017), for example, show results favoring their use, while Chkili (2021) and Sampid et al. (2018) show results contrary to this;

(iii) whether more complex statistical probability distributions provide more appropriate results than the normal distribution in predicting VaR.

The overall contribution of this paper is to assess all these points together using appropriate computational methods and to point out the most suitable statistical specification to be used in measuring the VaR of both energy and agricultural commodities.

This paper proceeds as follows: Section 2 presents the statistical specifications and methodology used, Section 3 exposes the database and discusses the empirical results found, and finally, conclusions are reported in Section 4.

2. Specification and methodology

The measurement and evaluation of VaR quality is performed in four steps according to a univariate parametric structure, namely:

(1) The statistical model that describes the dynamics of log-returns of time series is defined. In this research, we define 48 different specifications of the Markov-switching GARCH model of Haas et al. (2004), composed of four scedastic specifications, six conditional distributions, and a Markov chain with up to two regimes.

(2) The model parameters for a given estimation window are estimated and the distribution of day-ahead log-returns is calculated. Here, similar with the procedures adopted by Gerlach, Wang (2020), Wang et al. (2019) and Wu et al. (2020), we employ a rolling window approach, with a fixed sample size of 5,000 observations, to perform the parameter estimates at each prediction step. We set the total number of predictions for each model at 250, which represents 5% of the fixed size of the established rolling window. This choice was made arbitrarily, justifying itself

by presenting enough, without jeopardizing the parameter estimation period, to measure the forecast performance of the models. Thus, in total we made 48000 forecasts, consisting of 12000 (250×48) forecasts for each commodity.

(3) The VaR is measured from the distribution of log-returns calculated in the previous step. For this, we use the 99% confidence level that the Basel III Capital Accord of Basel Committee on Banking Supervision (2011) recommends that banks adopt and we also consider the 97.5% and 95% levels. With these three confidence levels, we total 144000 (48000×3) VaR estimates.

(4) The quality of the *ex post* VaR forecast is evaluated through statistical procedures, which are called backtesting. This subject is discussed in more detail in Section 2.3.

2.1. Markov-switching GARCH model specification

The implementation of the Markov-switching GARCH (MSGARCH) model requires the variable of interest y_t to assume the following moment conditions: $\mathrm{E}\left[y_t\right]=0$ and $\mathrm{E}\left[y_t y_{t-1}\right]=0$ for all $t>0$. Therefore, we define $y_t\in\mathbb{R}$ as the log-return of commodities at time *t*, with $y_t=\log\left(P_t/P_{t-1}\right)$, where P_t is the price at *t*, and assume that y_t has mean zero and is not serially correlated. With these conditions, the general specification of the model, according to (Ardia et al., 2018), can be expressed through the following mathematical expression:

$$y_t\,|\left(s_t=k,\mathcal{I}_{t-1}\right)\sim\mathcal{D}\left(0,h_{k,t},\xi_k\right), \tag{1}$$

where $\mathcal{I}_{t-1}$ is the set of information observed up to time $t-1$, that is, $\mathcal{I}_{t-1}=\left\{y_{t-i},i>0\right\}$, and $\mathcal{D}\left(0,h_{k,t},\xi_k\right)$ is a continuous distribution with zero mean, time-varying variance $h_{k,t}$ and additional shape parameters gathered in the vector ξ_k, such as tail and asymmetry. We also define the standardized innovations as $\eta_{k,t}=y_t\big/h_{k,t}^{1/2}\sim$ i.i.d. $\mathcal{D}\left(0,1,\xi_k\right)$. Furthermore, it is assumed that the latent variable s_t, defined in the discrete space $\left\{1,\ldots,K\right\}$, evolves according to an unobservable first-order homogeneous ergodic Markov chain with transition probability matrix $\mathbf{P}=\left\{p_{i,j}\right\}_{i,j=1}^{K}$, where $p_{i,j}=\Pr\left[s_t=j\,|s_{t-1}=i\right]$ is the transition probability from the state $s_{t-1}=i$ to $s_t=j$. In addition, $0<p_{i,j}<1$ $\forall i,j\in\left\{1,\ldots,K\right\}$ and $\sum_{j=1}^{K}p_{i,j}=1$, $\forall i\in\left\{1,\ldots,K\right\}$ are the restrictions of the model. Thus, given the parameterization of $\mathcal{D}(\cdot)$, we have $\mathrm{E}\left[y_t^2\,|s_t=k,\mathcal{I}_{t-1}\right]=h_{k,t}$, that is, $h_{k,t}$ is the conditional variance of y_t in the realization of $s_t=k$ and the information set $\mathcal{I}_{t-1}$. According to (Haas et al., 2004), the conditional variance of y_t is assumed to follow a GARCH process and thus, conditional on the regime $s_t=k$, $h_{k,t}$ is evaluated as a function of y_{t-1}, $h_{k,t-1}$ and θ_k, which represent, respectively, the past observations, the past variances and the additional regime-dependent vector of the parameters, according to the expression:

$$h_{k,t}=h\left(y_{t-1},h_{k,t-1},\,\theta_k\right), \tag{2}$$

where $h(\cdot)$ is a measurable function of $\mathcal{I}_{t-1}$, which defines the filter for the conditional variance and also guarantees its positivity. Furthermore, it is assumed that $h_{k,1}=\overline{h}_k$ $\left(k=1,\ldots,K\right)$, where $\overline{h}_k$ is a fixed unconditional variance level for regime *k*. Thus, depending on the specification of $h(\cdot)$, different scedastic specifications are obtained. Even single regime (SR) GARCH models can be obtained by means of $h(\cdot)$ when we set $k=1$. In this research, four different specifications for the conditional variance are considered. The first refers to the ARCH model of Engle (1982), given by:

$$h_{k,t} = \alpha_{0,k} + \alpha_{1,k} y_{t-1}^2, \tag{3}$$

to $k = 1, \ldots, K$. In this case, we have $\theta_k = \left(\alpha_{0,k}, \alpha_{1,k}\right)^{\top}$. $\alpha_{0,k} > 0$ and $\alpha_{1,k} > 0$ are the conditions imposed for positivity of the model and the stationary covariance of each regime is obtained assuming that $\alpha_{1,k} < 1$. The second specification refers to the GARCH model of Bollerslev (1986), given by:

$$h_{k,t} = \alpha_{0,k} + \alpha_{1,k} y_{t-1}^2 + \beta_k h_{k,t-1}, \tag{4}$$

to $k = 1, \ldots, K$. In this case, we have $\theta_k = \left(\alpha_{0,k}, \alpha_{1,k}, \beta_k\right)^{\top}$. $\alpha_{0,k} > 0$, $\alpha_{1,k} > 0$ and $\beta_k \geq 0$ are the conditions imposed for positivity of the model and the stationary covariance of each regime is obtained assuming that $\alpha_{1,k} + \beta_k < 1$. The third specification used by us refers to the eGARCH model of Nelson (1991), given by:

$$\log\left(h_{k,t}\right) = \alpha_{0,k} + \alpha_{1,k}\left(|\eta_{k,t-1}| - \mathrm{E}\left[|\eta_{k,t-1}|\right]\right) + \alpha_{2,k} y_{t-1} + \beta_k \log\left(h_{k,t-1}\right), \tag{5}$$

to $k = 1, \ldots, K$, where the expectation $\mathrm{E}\left[|\eta_{k,t-1}|\right]$ is given with respect to the distribution conditional on the k regime. In this case, we have $\theta_k = \left(\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \beta_k\right)^{\top}$. The parameters have no restrictions imposed on the positivity in the conditional variance, since the model uses a logarithmic specification to be estimated, preventing the variance from being negative. Already the stationary covariance of each regime is obtained assuming that $\beta_k < 1$. Finally, the fourth specification used refers to the gjrGARCH model of Glosten et al. (1993), given by:

$$h_{k,t} = \alpha_{0,k} + \left(\alpha_{1,k} + \alpha_{2,k} \mathrm{I}\left(y_{t-1} < 0\right)\right) y_{t-1}^2 + \beta_k h_{k,t-1}, \tag{6}$$

to $k = 1, \ldots, K$, where $\mathrm{I}(\cdot)$ is a binary indicator that takes the value one if the condition $(\cdot)$ is valid and zero otherwise. In this case, we have $\theta_k = \left(\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \beta_k\right)^{\top}$. The parameter $\alpha_{2,k}$ controls the degree of asymmetry for the conditional volatility response to the past shock in regime *k*. The positivity condition requires that $\alpha_{0,k} > 0$, $\alpha_{1,k} > 0$, $\alpha_{2,k} \geq 0$ and $\beta_k \geq 0$. Furthermore, the stationary covariance of each regime is obtained by requiring that $\alpha_{1,k} + \alpha_{2,k} \mathrm{E}\left[\eta_{k,t}^2 \mathrm{I}(\eta_{k,t} < 0)\right] + \beta_k < 1$.

Now, to complete the specifications of the models presented, we need to define the conditional distribution of the standardized innovations $\eta_{k,t}$. In this paper, we consider six different distributions, namely: the standard normal distribution (norm), the standardized Student-*t* distribution (std), the standardized generalized error distribution (ged), and the standardized skewed versions of norm, std and ged, denoted respectively by snorm, sstd and sged. The asymmetric versions are obtained following the procedures adopted by Bauwens, Laurent (2005) and Fernández, Steel (1998). Finally, we expose in this section the combinations performed to obtain our set of competing models consisting of 48 different specifications. We combine: (i) the number of regimes, $k \in \{1, 2\}$, where we obtain a single regime (SR) specification when $k = 1$, and Markov-switching (MS) when $k = 2$; (ii) the specification of the scedastic function, which we define as ARCH, GARCH, eGARCH and gjrGARCH; and (iii) the specification of the conditional distribution, $\mathcal{D} \in$ {norm, std, ged, snorm, sstd, sged}.

2.2. Model estimation

To estimate the MSGARCH model of equation (1), we first need to write its likelihood function by regrouping the model parameters into $\Psi = (\theta_1, \xi_1, \ldots, \theta_K, \xi_K, \mathbf{P})$. The conditional density of y_t in state $s_t = k$ given Ψ and $\mathcal{I}_{t-1}$ is denoted by $f\mathcal{D}(y_t \mid s_t = k, \Psi, \mathcal{I}_{t-1})$. Then, integrating the state variable s_t, we obtain the density of y_t in state $s_t = k$ given Ψ and $\mathcal{I}_{t-1}$ only:

$$f(y_t \mid \Psi, \mathcal{I}_{t-1}) = \sum_{i=1}^{K} \sum_{j=1}^{K} p_{i,j} \eta_{i,t-1} f\mathcal{D}(y_t \mid s_t = j, \Psi, \mathcal{I}_{t-1}), \tag{7}$$

where $\eta_{i,t-1} = \Pr[s_{t-1} = i \mid \Psi, \mathcal{I}_{t-1}]$ represents the filtered probability in state i at time $t-1$, obtained by means of Hamilton's filter. For more details, see Hamilton (1989) and Hamilton (1994, Chapter 22). With this, the likelihood function is obtained from equation (7) as follows:

$$\mathcal{L}(\Psi, \mathcal{I}_T) = \prod_{t=1}^{T} f(y_t \mid \Psi, \mathcal{I}_{t-1}), \tag{8}$$

where $f(y_t \mid \Psi, \mathcal{I}_{t-1})$ represents the density of y_t, given the past observations $\mathcal{I}_{t-1}$ and the model parameters Ψ. Therefore, to estimate the model parameters we use the maximum likelihood estimator $\hat{\Psi}$ obtained by maximizing the logarithm of equation (8).

2.3. VaR predictions and accuracy

The VaR specification, valid for any probability distribution function, discrete or continuous, can be expressed using probabilistic terms. Since y_t represents the log-returns of a financial asset at time t, the VaR_t^α in the $(1-\alpha)$ percentile is defined by $\Pr(y_t \le VaR_t^\alpha) = \alpha$, which calculates the probability that the log-return at time t is less than or equal to VaR_t^α, given a significance level $\alpha \in (0,1)$. In other words, VaR measures the threshold value such that the probability of observing a loss greater than or equal to it over a given time horizon is equal to α (see (McNeil et al., 2015) for more details). In this paper, we calculate VaR in its parametric form, using day-ahead volatility forecasts obtained by MSGARCH models. For this, we first compute the conditional probability density function (pdf) one step ahead of y_{t+1} as a mixture of K regime-dependent distributions:

$$f(y_{T+1} \mid \Psi, \mathcal{I}_T) = \sum_{k=1}^{K} \pi_{k,T+1} f\mathcal{D}(y_{T+1} \mid s_{T+1} = k, \Psi, \mathcal{I}_T), \tag{9}$$

where $\pi_{k,T+1} = \sum_{i=1}^{K} p_{i,k} \eta_{i,T}$ is the mixing weights with $\eta_{i,T} = \Pr[s_T = i \mid \Psi, \mathcal{I}_T]$ $(i = 1, \ldots, K)$ being the filtered probabilities at time T. With equation (9) we can obtain the cumulative density function (cdf) as follows:

$$f(y_{T+1} \mid \Psi, \mathcal{I}_T) = \int_{-\infty}^{y_{T+1}} f(z \mid \Psi, \mathcal{I}_T) dz. \tag{10}$$

Here, we compute equations (9) and (10) easily by replacing Ψ with the maximum likelihood estimator $\hat{\Psi}$. With this, we can perform the VaR forecast for $T+1$ at risk level α by:

$$VaR_{T+1}^{\alpha} = \inf\left\{ y_{T+1} \in \mathbb{R} \mid F\left(y_{T+1} \mid \mathcal{I}_T\right) = \alpha \right\}, \tag{11}$$

where $F\left(y \mid \mathcal{I}_T\right)$ is the one-step ahead cumulative density function (cdf) evaluated in y (see (Ardia et al., 2018) for more details on calculations). We use equation (11) to perform the VaR predictions in each competing heteroskedastic model considering risk levels of 1, 2.5 and 5%. After that, we conducted performance tests of the models, to analyze both the accuracy of the predictions and the accuracy of the left tail distribution. We start by calculating the proportion of returns in the forecast period that exceed the level of the forecasted VaR. For this, we use the VaR violation rate, defined by $VRate = \frac{1}{m}\sum_{t=n+1}^{n+m} \mathrm{I}\left(y_t < VaR_t^{\alpha}\right)$, where n is the in-sample size and m is the out-of-sample size. Models with *VRate* values closer to the level of the set nominal quantile α are preferred. Regarding models that present a *VRate* with the same absolute distances at the nominal quantile level, we follow Gerlach, Wang (2020) and Wang et al. (2019) and choose the conservative one as preferred. For example, 0.95% is preferred over 1.05%. Note that having a *VRate* close to α is a necessary but not sufficient condition for a predictive model to be accurate. Therefore, we complement this analysis by employing the following tests, commonly used in the literature: the unconditional coverage (UC) test of Kupiec (1995), the conditional coverage (CC) test of Christoffersen (1998), and the dynamic quantile (DQ) test of Engle, Manganelli (2004), with 4 lags, as did Maciel (2021). In addition, we follow Mcaleer, Da Veiga (2008) and calculate the mean absolute deviations (ADmean) between the observations and the quantiles, which provides a measure of the expected loss given a VaR violation. Models with lower ADmean are preferable. As if that were not enough, we also applied the quantile loss function (QL) of González-Rivera et al. (2004), which provides a weighted average of the difference in observed returns relative to the VaR value, giving greater weight to losses that violate the VaR level. Formally, given a VaR forecast at a risk level α for time *t*, QL is defined as: $QL_t^{\alpha} = \left(\alpha - I_t^{\alpha}\right)\left(y_t - VaR_t^{\alpha}\right)$, where $I_t^{\alpha} = \mathrm{I}(y_t < VaR_t^{\alpha})$. Moving forward, we create an overall average ranking of the competing model performances for all calculated measures and find a preferable model, which provides the best average forecast quality for all four commodities. Finally, we run the model confidence set (MCS) procedure proposed by Hansen et al. (2011) to create a set that contains the best models, which have statistically the same predictive ability, given a significance level, selected to be 10% here (as done by Wang et al. (2019) and Gerlach, Wang (2020)).

3. Data and empirical study

3.1. Datasets

The data used refers to daily closing prices of four commodity futures contracts, namely: Crude Oil WTI (Oil), Natural Gas (Gas), Corn (Corn) and Soybean Oil (Soy). The first two refer to non-renewable energies and their contracts are traded on the New York Mercantile Exchange (NYMEX). The last two refer to raw materials used in the production of biofuels and their contracts are traded on the Chicago Board of Trade (CBOT). All data series were collected from Investing.com[2] and is expressed in US dollars. The daily percentage log-return series for each commodity was calculated using $y_t = \log\left(P_t / P_{t-1}\right) \times 100$, where P_t represents the closing price at time *t*. The sample periods of the four assets and the descriptive statistics of the calculated log-returns are reported in Table 1.

Table 1. Descriptive statistics of the commodities log-return series

Measures	Oil	Gas	Corn	Soy
Mean*	0.0172	0.0137	0.0072	0.0092
Median*	0.0489	0.0000	0.0000	0.0000
Maximum	0.3196	0.3817	0.2503	0.0904
Minimum	–0.4005	–0.3757	–0.2762	–0.1102
Standard deviation*	2.4985	3.4706	1.6938	1.5011
Skewness	–0.5011	0.2720	–1.1263	–0.0100
Kurtosis	23.9179	12.1185	31.0846	5.5368
Jarque–Bera	179939.5	28120.2	352984.9	2861.5
Date				
Start	04/06/83	04/06/90	12/31/79	01/02/80
End	01/31/22	01/31/22	01/31/22	01/31/22
Size	9847	8088	10672	10671

Note. * — values were multiplied by 10^2.

All series exhibit leptokurtic distributions, as they exhibit asymmetry (mostly negative) and excess kurtosis. These characteristics are more pronounced in the Corn, Oil and Gas series, and more moderate in the Soy series, but the heavy tails are present in all series. This means that large negative returns are more likely to occur and that if we estimate the VaR under the assumption of normality, we may be underestimating it. This evidence of non-normality is confirmed by the Jarque–Bera test. To complement this analysis, we expose the graphical representations of the log-returns in Fig. 1. Here some information is important to mention:

(a) the returns on non-renewable energy commodities have higher extreme values than those on renewable energy ones;

(b) the Gas and Oil series have the highest annualized volatilities ($\text{Standard deviation} \times \sqrt{250} \times 100$), 54.88% and 39.50% respectively; and

(c) volatility clusters are present in all commodities, which is usually observed in log-returns of traditional financial asset prices.

[2] See https://www.investing.com/.

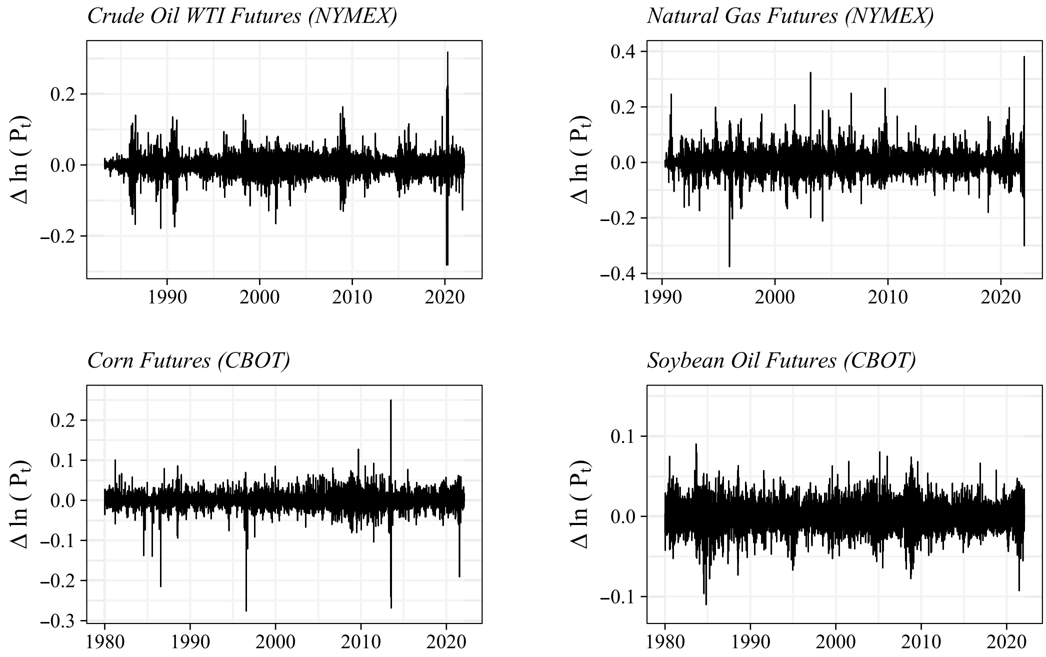

Fig. 1. Log-returns of commodity prices for the sample period

3.2. Empirical study

We have run the proposed method and here summarize the results with respect to our main research purpose: to find a common statistical specification to reliably estimate the VaR for all selected commodities. If necessary, detailed test estimates can be found in Appendix A. We begin by exploring Table 2, which shows the count of times each model showed VRate values closest to the level of nominal quantile set across all four commodities. Results can be summarized in two findings.

(i) Markov chain. In general, single regime models are preferred, although the preference for MS models increases as the level of nominal quantile increases.

(ii) Scedastic specifications and distributions. Excluding the ARCH specification which performed poorly, all other scedastic specifications showed good performance when combined with the distributions std, sstd, ged and sged.

Moving on, we explore Table 3, which shows the count of rejections of the null hypothesis of correct VaR prediction for the UC, CC and DQ tests across all four commodities. Models with lower rejections are preferable. We also analyze this result at two points.

(i) Markov chain. Single regime models are more numerous among the preferable ones, although there is no such pronounced predominance. We also observe that as the nominal quantile level increases, the total number of rejections decreases more significantly among the MS models.

(ii) Scedastic functions and distributions. Here we notice better performance of the gjrGARCH and GARCH scedastic functions and again poor performance of ARCH. On the distributions, we could not visualize any significant preference, although the std, sstd, ged and sged distributions appear most frequently among the preferred specifications.

Table 2. Number of times the model obtained a better *VRate* across the four commodities

Model	α = 1%		α = 2.5%		α = 5%		Total	
	SR	MS	SR	MS	SR	MS	SR	MS
ARCH-norm	0	**1**	0	0	0	0	0	1
ARCH-snorm	0	0	1	**2**	0	0	1	2
ARCH-std	0	**1**	0	1	0	**1**	0	3
ARCH-sstd	0	**1**	0	1	0	**1**	0	3
ARCH-ged	0	**1**	1	1	0	**1**	1	3
ARCH-sged	0	**1**	1	1	0	0	1	2
GARCH-norm	0	0	1	1	**1**	0	2	1
GARCH-snorm	0	0	1	**2**	**1**	0	2	2
GARCH-std	[2]	**1**	**2**	1	**1**	**1**	**5**	3
GARCH-sstd	**1**	0	**2**	**2**	**1**	**1**	4	3
GARCH-ged	**1**	0	**2**	1	0	[2]	3	3
GARCH-sged	0	0	[3]	1	**1**	0	4	1
eGARCH-norm	**1**	0	1	1	0	0	2	1
eGARCH-snorm	0	**1**	0	1	0	0	0	2
eGARCH-std	[2]	**1**	**2**	**2**	0	0	4	3
eGARCH-sstd	[2]	0	**2**	1	0	**1**	4	2
eGARCH-ged	[2]	**1**	**2**	**2**	0	**1**	4	4
eGARCH-sged	[2]	0	**2**	1	0	[2]	4	3
gjrGARCH-norm	0	0	1	1	[2]	0	3	1
gjrGARCH-snorm	0	0	1	**2**	**1**	**1**	2	3
gjrGARCH-std	[2]	0	**2**	**2**	[2]	0	[6]	2
gjrGARCH-sstd	**1**	0	1	**2**	0	0	2	2
gjrGARCH-ged	**1**	0	**2**	0	**1**	[2]	4	2
gjrGARCH-sged	0	0	**2**	0	**1**	**1**	3	1
Total	17	9	32	29	12	15	61	53

Note. Boxes indicate the favored model and bold indicates the second-ranked model.

We follow with Table 4, which presents the rankings of the models that have the lowest mean absolute deviations (ADmean) between the observations and the quantiles across all four commodities. In the overall average ranking, MS models occupy the two positions that have the lowest measure of expected loss given a VaR violation. Exploring the levels individually, we note that at the 1% level the top two positions belong to single regime models, while at the 2.5% and 5% level these positions belong to MS models. Furthermore, in the results of this comparison, we observe that most of the models listed as preferable, both in the individual risk levels and the overall average, use norm and snorm distributions. Here we find a divergence with the previous results. As for the scedastic functions, we could not visualize any significant preference pattern.

We complement these results with Table 5, which shows the rankings of the models with the lowest quantile loss function (QL) values across all four commodities. We explore this result also at two points.

(i) Markov chain. The models selected as preferred are mostly single regime, both in the individual analysis of each level of risk and in the analysis of the overall average.

Table 3. Counts of VaR rejections with UC, CC and DQ tests across the four commodities

Model	$\alpha = 1\%$		$\alpha = 2.5\%$		$\alpha = 5\%$		Total	
	SR	MS	SR	MS	SR	MS	SR	MS
ARCH-norm	6	$\boxed{4}$	6	4	4	1	16	**9**
ARCH-snorm	6	6	6	5	4	1	16	12
ARCH-std	**5**	$\boxed{4}$	6	**3**	5	**2**	16	**9**
ARCH-sstd	**5**	**5**	7	**3**	5	3	17	11
ARCH-ged	**5**	$\boxed{4}$	5	4	5	$\boxed{1}$	15	**9**
ARCH-sged	**5**	$\boxed{4}$	6	5	5	4	16	13
GARCH-norm	$\boxed{4}$	**5**	**3**	**3**	$\boxed{1}$	$\boxed{1}$	$\boxed{8}$	**9**
GARCH-snorm	$\boxed{4}$	**5**	**3**	**3**	$\boxed{1}$	$\boxed{1}$	$\boxed{8}$	**9**
GARCH-std	$\boxed{4}$	**5**	**3**	**3**	$\boxed{1}$	$\boxed{1}$	$\boxed{8}$	**9**
GARCH-sstd	**5**	**5**	**3**	**3**	$\boxed{1}$	$\boxed{1}$	**9**	**9**
GARCH-ged	$\boxed{4}$	$\boxed{4}$	**3**	**3**	$\boxed{1}$	$\boxed{1}$	$\boxed{8}$	$\boxed{8}$
GARCH-sged	**5**	**5**	**3**	**3**	$\boxed{1}$	$\boxed{1}$	**9**	**9**
eGARCH-norm	6	6	**3**	**3**	$\boxed{1}$	$\boxed{1}$	10	10
eGARCH-snorm	6	6	**3**	**3**	$\boxed{1}$	$\boxed{1}$	10	10
eGARCH-std	**5**	6	**3**	**3**	$\boxed{1}$	$\boxed{1}$	**9**	10
eGARCH-sstd	**5**	$\boxed{4}$	**3**	**3**	**2**	$\boxed{1}$	10	$\boxed{8}$
eGARCH-ged	**5**	**5**	**3**	**3**	$\boxed{1}$	$\boxed{1}$	**9**	**9**
eGARCH-sged	**5**	$\boxed{4}$	**3**	**3**	$\boxed{1}$	$\boxed{1}$	**9**	$\boxed{8}$
gjrGARCH-norm	$\boxed{4}$	$\boxed{4}$	**3**	**3**	$\boxed{1}$	$\boxed{1}$	$\boxed{8}$	$\boxed{8}$
gjrGARCH-snorm	$\boxed{4}$	**5**	**3**	**3**	$\boxed{1}$	$\boxed{1}$	$\boxed{8}$	**9**
gjrGARCH-std	$\boxed{4}$	**5**	**3**	**3**	$\boxed{1}$	$\boxed{1}$	$\boxed{8}$	**9**
gjrGARCH-sstd	$\boxed{4}$	**5**	**3**	$\boxed{2}$	$\boxed{1}$	$\boxed{1}$	$\boxed{8}$	$\boxed{8}$
gjrGARCH-ged	$\boxed{4}$	$\boxed{4}$	**3**	**3**	$\boxed{1}$	$\boxed{1}$	$\boxed{8}$	$\boxed{8}$
gjrGARCH-sged	$\boxed{4}$	**5**	**3**	**3**	$\boxed{1}$	$\boxed{1}$	$\boxed{8}$	**9**
Total	114	115	90	77	47	30	251	222

Note. Boxes indicate the models with the fewest rejections and bold indicates the models with the second-lowest numbers of rejections. All tests are conducted at the 10% significance level.

(ii) Scedastic functions and distributions. Clearly, the GARCH scedastic function provides the lowest value of QL in all scenarios. In addition, we note a pattern of preference of the ged and sged distributions for almost all specifications in the accuracy of the left-tailed return prediction.

We have now compiled all these results into an overall ranking, which is shown in Table 6. We also analyze this result in two points.

(i) Markov chain. We find that at the 1, 2.5% and overall average risk level the preferable models are single regime, while at the 5% risk level the preferable ones are MS. The preference for MS increases when we increase the level of risk.

(ii) Scedastic functions and distributions. The models listed as preferred use either the GARCH or the gjrGARCH model, in all scenarios. Regarding the choice of probability distribution, we notice better overall performances when the std and ged distributions are selected. Overall, in the search for a preferred pattern in estimating VaR for all commodities and at all risk levels, the gjrGARCH-std and GARCH-ged models stand out.

Table 4. Ranking of ADmean positions of returns that violate the VaR forecasting across the four commodities

Model	α = 1%		α = 2.5%		α = 5%		Avg rank	
	SR	MS	SR	MS	SR	MS	SR	MS
ARCH-norm	34.50	35.75	29.00	07.25	22.00	13.25	28.50	**18.75**
ARCH-snorm	33.25	25.25	32.75	17.00	21.25	20.50	29.08	20.92
ARCH-std	27.25	33.00	29.00	24.50	19.00	23.50	25.08	27.00
ARCH-sstd	27.75	25.25	28.75	29.25	22.25	21.00	26.25	25.17
ARCH-ged	27.50	33.25	36.25	23.75	20.75	25.50	28.17	27.50
ARCH-sged	27.50	29.75	26.75	27.75	22.50	25.50	25.58	27.67
GARCH-norm	19.00	19.75	23.00	25.75	27.00	**12.00**	23.00	19.17
GARCH-snorm	**15.00**	20.25	26.25	26.50	26.75	18.25	22.67	21.67
GARCH-std	23.75	31.00	25.50	17.00	25.00	30.00	24.75	26.00
GARCH-sstd	26.75	19.25	30.25	21.50	29.00	25.50	28.67	22.08
GARCH-ged	25.25	26.00	14.75	19.75	38.50	20.50	26.17	22.08
GARCH-sged	29.75	21.50	28.00	**12.75**	39.25	28.75	32.33	21.00
eGARCH-norm	20.50	23.25	24.25	22.75	24.00	15.25	22.92	20.42
eGARCH-snorm	12.25	22.00	25.75	35.75	29.00	14.75	22.33	24.17
eGARCH-std	23.00	17.00	26.50	23.00	27.50	24.50	25.67	21.50
eGARCH-sstd	26.25	16.50	36.75	31.75	37.00	23.25	33.33	23.83
eGARCH-ged	22.50	27.75	23.50	21.00	24.00	21.75	23.33	23.50
eGARCH-sged	25.00	28.50	30.25	31.25	33.25	22.50	29.50	27.42
gjrGARCH-norm	21.75	19.25	17.75	27.00	24.50	13.75	21.33	20.00
gjrGARCH-snorm	19.25	17.25	21.75	26.00	26.50	11.25	22.50	18.17
gjrGARCH-std	25.50	28.75	20.25	20.00	24.50	28.00	23.42	25.58
gjrGARCH-sstd	21.50	16.75	25.25	25.75	23.75	23.00	23.50	21.83
gjrGARCH-ged	27.50	31.00	15.75	22.50	35.75	25.00	26.33	26.17
gjrGARCH-sged	22.25	21.25	22.75	13.50	34.75	30.00	26.58	21.58

Note. Boxes indicate the favored models and bold indicates the second-ranked model.

Finally, the MCS procedure is run. Table 7 exposes the count of the number of inclusions of each model in the sets established by the procedure for all four commodities. The greater the number of inclusions, the better performance the model has. Exploring these results in more detail, we note that the GARCH-ged model, which stood out in the previous analyses, has the highest number of ensemble inclusions. This means that, most of the time, this statistical specification is among those that performed best in the scenarios outlined and that have statistically the same predictive ability. Thus, in the search for a statistical specification that has the best VaR forecast quality for all commodities, we strongly advise choosing GARCH-ged. This choice favors the use of a single regime model, reinforcing the results found in this regard by Chkili (2021) and Sampid et al. (2018) and contradicting the results of Ardia et al. (2018, 2019), Caporale, Zekokh (2019) and Segnon et al. (2017). At this point, we point out that we notice an increase in preference for MS models when we increase the level of the nominal quantile. This may perhaps explain these possible divergences in results in the literature.

Table 5. Ranking of QL positions for VaR forecasting across the four commodities

Model	α = 1%		α = 2.5%		α = 5%		Avg rank	
	SR	MS	SR	MS	SR	MS	SR	MS
ARCH-norm	45.25	26.50	40.00	18.25	32.25	25.00	39.17	23.25
ARCH-snorm	44.50	31.25	39.00	22.50	33.25	25.75	38.92	26.50
ARCH-std	35.50	28.50	38.25	29.50	42.25	39.00	38.67	32.33
ARCH-sstd	36.25	31.75	37.50	32.25	38.75	38.75	37.50	34.25
ARCH-ged	36.50	29.25	33.75	17.00	35.00	27.00	35.08	24.42
ARCH-sged	36.25	26.50	34.50	24.50	35.00	32.25	35.25	27.75
GARCH-norm	21.50	25.50	20.25	27.50	10.25	28.50	17.33	27.17
GARCH-snorm	23.75	25.75	22.75	25.50	12.75	27.75	19.75	26.33
GARCH-std	09.75	13.00	19.00	16.75	21.75	21.75	16.83	17.17
GARCH-sstd	14.00	13.75	20.50	**14.00**	26.00	16.75	20.17	14.83
GARCH-ged	**11.00**	19.75	13.00	**14.00**	10.25	16.50	11.42	16.75
GARCH-sged	12.75	20.50	**14.00**	22.25	13.00	18.75	**13.25**	20.50
eGARCH-norm	32.50	26.00	27.75	26.75	19.25	28.75	26.50	27.17
eGARCH-snorm	37.50	28.75	32.25	27.75	24.00	32.25	31.25	29.58
eGARCH-std	19.50	24.00	24.75	28.75	29.25	30.25	24.50	27.67
eGARCH-sstd	21.00	26.50	29.75	30.75	35.25	31.75	28.67	29.67
eGARCH-ged	17.25	23.00	18.25	23.25	17.00	24.75	17.50	23.67
eGARCH-sged	20.00	28.75	23.75	27.00	23.50	26.00	22.42	27.25
gjrGARCH-norm	23.50	25.50	22.50	29.25	11.75	26.25	19.25	27.00
gjrGARCH-snorm	27.75	29.25	24.75	22.50	13.75	23.25	22.08	25.00
gjrGARCH-std	15.50	14.50	20.00	19.50	21.25	16.00	18.92	16.67
gjrGARCH-sstd	16.25	15.50	21.75	20.25	27.75	19.25	21.92	18.33
gjrGARCH-ged	15.75	29.00	14.75	25.25	**11.50**	23.00	14.00	25.75
gjrGARCH-sged	17.25	22.50	16.00	22.25	14.25	17.75	15.83	20.83

Note. Boxes indicate the favored models and bold indicates the second-ranked model; QL function is conducted at the 1% significance level.

As a practical example, we plot Fig. 2 to graphically analyze the performance of the proposed specification in predicting VaR at the 5% risk level. We make this exercise more interesting by also plotting the predictions of a competing model for comparison. Here, we have chosen the MSGARCH-sged model, which was the best performer in this individual scenario (Oil series and 5% risk level). As we can see, the proposed statistical specification accomplishes its purpose very well.

4. Conclusion

Changes in commodity prices can be transmitted directly to the real economy through changes in the marginal cost of production. These movements can cause important losses for economic agents who are not prepared to face them. For this reason, it is of utmost importance to create protection mechanism against these movements. One way to protect itself is to take exposure in the commodities futures market, but this involves risks that must be considered by economic

Table 6. Overall ranking of the average positions on each statistical measure

Model	α = 1%		α = 2.5%		α = 5%		Avg rank	
	SR	MS	SR	MS	SR	MS	SR	MS
ARCH-norm	39.25	21.25	43.00	22.75	29.50	13.00	37.25	19.00
ARCH-snorm	38.50	31.50	39.00	17.25	29.75	14.75	35.75	21.17
ARCH-std	28.75	21.25	42.50	20.75	31.00	28.25	34.08	23.42
ARCH-sstd	30.00	23.00	43.00	25.75	32.25	26.00	35.08	24.92
ARCH-ged	29.75	22.50	37.75	22.00	30.25	16.75	32.58	20.42
ARCH-sged	29.25	19.25	35.25	30.50	31.75	33.25	32.08	27.67
GARCH-norm	12.00	19.25	13.50	20.25	10.75	14.75	12.08	18.08
GARCH-snorm	11.75	20.00	18.75	16.25	11.50	15.50	14.00	17.25
GARCH-std	[06.50]	18.25	10.00	08.75	13.25	16.50	09.92	14.50
GARCH-sstd	16.00	13.50	15.25	04.75	18.25	12.00	16.50	10.08
GARCH-ged	08.75	16.50	[02.25]	08.25	18.00	[05.25]	**09.67**	10.00
GARCH-sged	21.00	18.25	10.25	10.50	15.25	19.00	15.50	15.92
eGARCH-norm	25.50	28.00	19.50	17.25	15.50	16.00	20.17	20.42
eGARCH-snorm	27.25	25.00	27.75	25.50	21.50	16.75	25.50	22.42
eGARCH-std	14.00	19.25	15.50	14.50	23.75	21.25	17.75	18.33
eGARCH-sstd	17.25	13.50	22.75	26.50	38.00	16.00	26.00	18.67
eGARCH-ged	13.25	21.00	08.25	10.00	14.75	11.25	12.08	14.08
eGARCH-sged	15.25	23.50	17.50	24.25	22.25	11.50	18.33	19.75
gjrGARCH-norm	15.25	13.75	12.50	23.25	07.75	14.25	11.83	17.08
gjrGARCH-snorm	15.50	21.00	15.75	13.25	11.75	**07.25**	14.33	13.83
gjrGARCH-std	09.50	21.75	07.00	06.50	11.00	17.50	[09.17]	15.25
gjrGARCH-sstd	**08.50**	13.25	15.75	10.75	19.25	14.50	14.50	12.83
gjrGARCH-ged	12.75	25.00	**03.50**	22.00	13.75	12.50	10.00	19.83
gjrGARCH-sged	13.25	18.75	06.50	16.25	14.75	15.25	11.50	16.75

Note. Boxes indicate the favored models and bold indicates the second-ranked model.

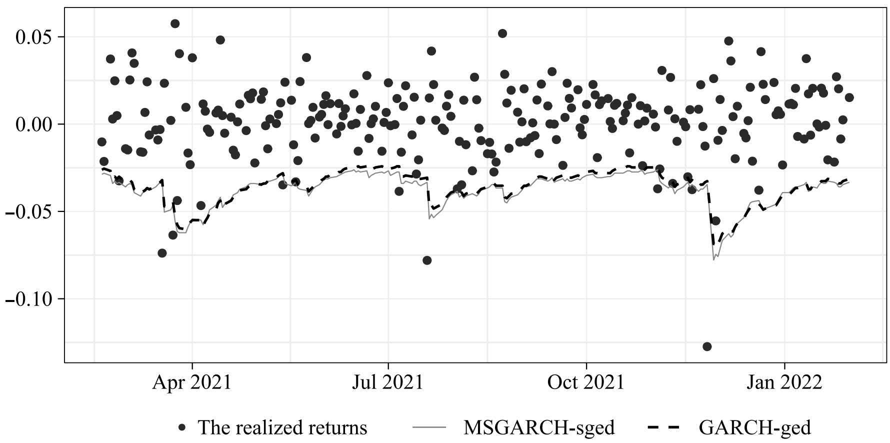

Fig. 2. One-day ahead VaR forecasts for commodity Oil at the 5% risk level provided by GARCH-ged and MSGARCH-sged, along with the realized returns

Table 7. Counts total number of inclusions in the MCS at 10% significance level

Model	α = 1%		α = 2.5%		α = 5%		Total	
	SR	MS	SR	MS	SR	MS	SR	MS
ARCH-norm	**2**	**2**	**2**	**2**	2	2	6	6
ARCH-snorm	**2**	**2**	**2**	**2**	1	**3**	5	7
ARCH-std	1	**2**	1	**2**	1	0	3	4
ARCH-sstd	1	1	1	**2**	1	1	3	4
ARCH-ged	1	**2**	**2**	$\boxed{3}$	1	1	4	6
ARCH-sged	1	**2**	**2**	**2**	1	1	4	5
GARCH-norm	**2**	**2**	1	1	$\boxed{4}$	0	7	3
GARCH-snorm	**2**	**2**	**2**	1	**3**	0	7	3
GARCH-std	$\boxed{3}$	**2**	1	**2**	1	2	5	6
GARCH-sstd	$\boxed{3}$	$\boxed{3}$	**2**	1	1	2	6	6
GARCH-ged	$\boxed{3}$	**2**	$\boxed{3}$	**2**	$\boxed{4}$	2	$\boxed{10}$	6
GARCH-sged	$\boxed{3}$	**2**	$\boxed{3}$	**2**	**3**	1	**9**	5
eGARCH-norm	1	1	1	**2**	**3**	1	5	4
eGARCH-snorm	1	1	1	1	2	1	4	3
eGARCH-std	$\boxed{3}$	**2**	1	**2**	1	1	5	5
eGARCH-sstd	$\boxed{3}$	1	1	1	0	1	4	3
eGARCH-ged	$\boxed{3}$	**2**	**2**	1	**3**	0	8	3
eGARCH-sged	$\boxed{3}$	**2**	**2**	1	2	1	7	4
gjrGARCH-norm	**2**	**2**	1	1	**3**	0	6	3
gjrGARCH-snorm	**2**	**2**	**2**	**2**	**3**	2	7	6
gjrGARCH-std	$\boxed{3}$	$\boxed{3}$	1	$\boxed{3}$	1	2	5	8
gjrGARCH-sstd	$\boxed{3}$	$\boxed{3}$	**2**	**2**	1	2	6	7
gjrGARCH-ged	$\boxed{3}$	**2**	$\boxed{3}$	**2**	**3**	2	**9**	6
gjrGARCH-sged	$\boxed{3}$	**2**	**2**	**2**	**3**	1	8	5

Note. Boxes indicate the favored model and bold indicates the second-ranked model, based on the total number of inclusions in the MCS across the four commodities.

agents, such as tail risk. Controlling this risk is essential within active risk management and can be accomplished by estimating VaR. Because it involves a certain complexity, economic agents may be facing difficulties in making reliable commodity VaR forecasts. For this and other reasons, this research sought to provide a common statistical specification for agents to reliably estimate the VaR of four important energy commodities. To this end, we analyze 48 different specifications of the Markov-switching GARCH model, consisting of four scedastic specifications, six conditional distributions, and a Markov chain with up to two regimes.

As a result of our empirical study, we proposed the GARCH-ged model as the common statistical specification. Here some observations can be made:

(i) Markov chain. A single regime model was preferred, showing that the use of Markov chains in estimating the models are not advisable for predicting the VaR of all four commodities studied.

(ii) Scedastic functions and distributions. The standard GARCH scedastic function was preferred, showing that a simple mathematical structure outperformed more complex mathematical structures that include several statistical parameters. Over the distributions, we note a pattern of preference of the ged distribution in the accuracy of the left-tailed return prediction. The choice of the right distribution really impacts the VaR estimation, and more complex distributions are preferable. We strongly advise against using the normal distribution for VaR estimation.

These results contribute to a better efficiency in the creation of protection mechanisms against significant oscillations in the prices of energy commodities. Thus, companies dependent on these commodities, both directly and indirectly, can gain competitive advantages over their competitors. Individuals who include these commodities in their consumption baskets can maximize their utility functions. In addition, companies, farmers, investors, investment funds, and the like can also benefit from these results to reduce the market risk of their conventional financial positions, since commodity prices tend to behave inversely to economic activity.

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Appendix A
Testing model performance in VaR forecasts

We present here the detailed estimation results of all statistical tests for a 1% significance level[3], as follows: Table A1 show the *VRate* values; Table A2 show the count of rejections of the null hypothesis of correct VaR prediction for the UC, CC and DQ tests; Table A3 show the values of mean absolute deviation of returns that violate the VaR forecasting; Table A4 expose the quantile loss function values for VaR forecasting; and finally, Table A5 present the model confidence set at 10% significance level for VaR forecasts.

Table A1. VaR violation rate values for 1% VaR forecasting

Model	Oil		Gas		Corn		Soy		Total	
	SR	MS	SR	MS	SR	MS	SR	MS	SR	MS
ARCH-norm	1.60	**1.20**	2.00	1.60	**2.40**	1.20	**4.00**	1.60	0	1
ARCH-snorm	1.60	**1.20**	2.00	**2.40**	**2.40**	1.20	**4.40**	**2.80**	0	0
ARCH-std	1.60	**1.20**	2.00	1.60	2.00	1.20	**3.60**	1.60	0	1
ARCH-sstd	1.60	1.60	2.00	**2.40**	2.00	1.20	**3.60**	1.60	0	1
ARCH-ged	1.60	**1.20**	2.00	1.20	2.00	1.20	**3.60**	1.60	0	1
ARCH-sged	1.60	1.60	2.00	1.60	2.00	1.20	**3.60**	1.60	0	1
GARCH-norm	2.00	2.00	1.20	1.60	2.00	2.00	**3.20**	**2.40**	0	0
GARCH-snorm	2.00	2.00	2.00	2.00	2.00	1.60	**3.20**	**2.40**	0	0
GARCH-std	**2.00**	**1.20**	0.80	1.20	0.80	0.80	**2.80**	**2.80**	2	1
GARCH-sstd	1.20	**1.20**	1.20	2.00	1.20	1.60	**3.20**	**2.40**	1	0
GARCH-ged	2.00	1.60	0.80	1.20	0.40	1.60	**3.20**	**2.40**	1	0
GARCH-sged	**1.20**	**1.20**	1.20	2.00	0.40	2.00	**3.20**	**2.40**	0	0
eGARCH-norm	2.80	2.80	0.80	1.20	2.00	1.20	**3.20**	**2.40**	1	0
eGARCH-snorm	2.80	2.80	2.00	1.60	2.00	0.80	**3.20**	**2.40**	0	1
eGARCH-std	2.40	2.80	0.80	0.80	0.80	1.60	**3.20**	**2.40**	2	1
eGARCH-sstd	2.40	2.00	0.80	2.00	0.80	1.60	**3.20**	**2.40**	2	0
eGARCH-ged	2.40	2.40	0.80	1.20	0.80	0.80	**3.20**	**2.40**	2	1
eGARCH-sged	2.40	2.00	0.80	1.60	0.80	1.20	**3.20**	**2.40**	2	0
gjrGARCH-norm	2.00	2.00	1.20	1.60	1.60	2.00	**3.20**	**2.40**	0	0
gjrGARCH-snorm	2.00	2.00	2.00	**2.40**	1.60	1.60	**3.20**	**2.40**	0	0
gjrGARCH-std	2.00	**1.20**	0.80	1.20	0.80	1.20	**2.80**	**2.40**	2	0
gjrGARCH-sstd	2.00	**1.20**	1.20	1.60	0.80	2.00	**3.20**	**2.40**	1	0

[3] The detailed estimation results of statistical tests for the other significance levels can be requested from the authors.

End of Table A1

Model	Oil		Gas		Corn		Soy		Total	
	SR	MS	SR	MS	SR	MS	SR	MS	SR	MS
gjrGARCH-ged	2.00	1.60	0.80	1.20	0.40	1.60	**2.80**	**2.40**	1	0
gjrGARCH-sged	2.00	[**1.20**]	1.20	2.00	0.40	2.00	**3.20**	**2.40**	0	0
Total	1	0	9	1	7	3	0	5	17	9

Note. Boxes indicate the preferred models; Bold indicates that the violation rate is significantly different from 1% according to the UC test. The test is conducted at the 10% significance level.

Table A2. Counts of 1% VaR rejections with UC, CC and DQ tests

Model	UC		CC		DQ		Total	
	SR	MS	SR	MS	SR	MS	SR	MS
ARCH-norm	2	1	1	1	3	2	6	[4]
ARCH-snorm	2	3	1	1	3	2	6	6
ARCH-std	1	1	1	1	3	2	5	[4]
ARCH-sstd	1	1	1	1	3	3	5	5
ARCH-ged	1	1	1	1	3	2	5	[4]
ARCH-sged	1	0	1	1	3	3	5	[4]
GARCH-norm	1	1	1	1	2	3	[4]	5
GARCH-snorm	1	1	1	1	2	3	[4]	5
GARCH-std	1	2	1	1	2	2	[4]	5
GARCH-sstd	2	2	1	1	2	2	5	5
GARCH-ged	1	1	1	1	2	2	[4]	[4]
GARCH-sged	2	2	1	1	2	2	5	5
eGARCH-norm	1	1	2	2	3	3	6	6
eGARCH-snorm	1	1	2	2	3	3	6	6
eGARCH-std	1	1	1	2	3	3	5	6
eGARCH-sstd	1	1	1	1	3	2	5	[4]
eGARCH-ged	1	1	1	1	3	3	5	5
eGARCH-sged	1	1	1	1	3	2	5	[4]
gjrGARCH-norm	1	1	1	1	2	2	[4]	[4]
gjrGARCH-snorm	1	2	1	1	2	2	[4]	5
gjrGARCH-std	1	2	1	1	2	2	[4]	5
gjrGARCH-sstd	1	2	1	1	2	2	[4]	5
gjrGARCH-ged	1	1	1	1	2	2	[4]	[4]
gjrGARCH-sged	1	2	1	1	2	2	[4]	5

Note. Boxes indicate the models with the fewest rejections. All tests are conducted at the 10% significance level.

Table A3. Mean absolute deviation of returns that violate the 1% VaR forecasting

Model	Oil		Gas		Corn		Soy		Avg rank	
	SR	MS	SR	MS	SR	MS	SR	MS	SR	MS
ARCH-norm	3.684	4.975	3.978	3.115	3.607	5.088	1.579	1.727	34.50	35.75
ARCH-snorm	3.602	4.844	4.254	2.632	3.495	4.883	1.499	0.997	33.25	25.25
ARCH-std	2.980	4.735	3.389	3.432	3.701	4.976	1.389	1.220	27.25	33.00
ARCH-sstd	2.686	3.618	3.617	2.790	3.768	4.995	1.505	1.156	27.75	25.25
ARCH-ged	2.980	4.691	3.385	3.441	3.701	4.679	1.406	1.229	27.50	33.25
ARCH-sged	2.735	3.471	3.577	3.680	3.701	4.642	1.508	1.159	27.50	29.75
GARCH-norm	3.072	2.904	3.390	3.217	2.741	2.852	1.038	1.240	19.00	19.75
GARCH-snorm	2.915	2.786	2.472	2.862	2.718	3.394	1.206	1.322	**15.00**	20.25
GARCH-std	2.784	4.298	4.794	3.827	5.679	6.125	0.910	0.917	23.75	31.00
GARCH-sstd	4.233	3.753	3.573	2.563	3.866	3.095	0.991	1.143	26.75	19.25
GARCH-ged	2.758	3.399	4.505	3.591	11.301	3.230	0.807	1.224	25.25	26.00
GARCH-sged	4.189	3.819	3.342	2.453	11.301	**2.651**	1.003	1.451	29.75	21.50
eGARCH-norm	2.491	2.500	6.294	4.290	2.897	4.196	1.038	1.135	20.50	23.25
eGARCH-snorm	2.323	2.341	3.092	3.293	2.885	6.209	1.210	1.205	12.25	22.00
eGARCH-std	2.605	**2.332**	6.158	6.266	5.853	3.249	0.782	0.921	23.00	17.00
eGARCH-sstd	2.365	2.954	6.631	3.083	5.965	3.305	0.961	1.008	26.25	16.50
eGARCH-ged	2.583	2.698	5.771	3.763	5.853	6.320	**0.784**	1.085	22.50	27.75
eGARCH-sged	2.343	2.967	6.243	3.528	5.881	4.256	0.978	1.242	25.00	28.50
gjrGARCH-norm	3.137	2.889	3.390	3.110	3.439	2.807	1.033	1.260	21.75	19.25
gjrGARCH-snorm	2.980	2.875	2.460	2.364	3.439	3.380	1.213	1.225	19.25	17.25
gjrGARCH-std	2.875	4.342	4.794	3.807	5.707	4.438	0.920	0.937	25.50	28.75
gjrGARCH-sstd	2.627	3.884	3.573	3.343	5.791	2.651	0.986	0.927	21.50	16.75
gjrGARCH-ged	2.849	3.317	4.538	6.257	11.356	3.285	0.920	1.235	27.50	31.00
gjrGARCH-sged	2.640	3.928	3.342	**2.440**	11.356	2.707	0.995	1.345	22.25	21.25

Note. All values were multiplied by 10^2; Boxes indicate the favored models; bold indicates the second-ranked model.

Table A4. Quantile loss function values for 1% VaR forecasting at 1% significance level

Model	Oil		Gas		Corn		Soy		Avg rank	
	SR	MS	SR	MS	SR	MS	SR	MS	SR	MS
ARCH-norm	1.162	1.162	1.607	1.302	1.320	1.107	1.012	0.757	45.250	26.500
ARCH-snorm	1.160	1.156	1.626	1.391	1.306	1.104	1.034	0.760	44.500	31.250
ARCH-std	1.116	1.155	1.579	1.369	1.252	1.109	0.917	0.714	35.500	28.500
ARCH-sstd	1.103	1.183	1.596	1.451	1.259	1.109	0.948	**0.711**	36.250	31.750
ARCH-ged	1.119	1.170	1.576	1.400	1.252	1.083	0.921	0.713	36.500	29.250
ARCH-sged	1.111	1.161	1.588	1.369	1.252	1.080	0.948	0.709	36.250	26.500
GARCH-norm	1.137	1.121	1.220	1.340	1.045	1.100	0.811	0.775	21.500	25.500
GARCH-snorm	1.128	1.112	1.267	1.368	1.042	1.091	0.847	0.782	23.750	25.750
GARCH-std	1.119	1.100	1.260	1.312	1.006	1.019	0.757	0.759	9.750	13.000
GARCH-sstd	1.103	1.095	1.262	1.331	1.007	1.009	0.800	0.770	14.000	13.750
GARCH-ged	1.116	1.128	1.244	1.275	1.008	1.040	0.760	0.783	**11.000**	19.750
GARCH-sged	1.100	1.097	1.244	1.291	1.007	1.046	0.804	0.836	12.750	20.500
eGARCH-norm	1.194	1.218	1.317	1.360	1.052	1.034	0.812	0.756	32.500	26.000

End of Table A4

Model	Oil		Gas		Corn		Soy		Avg rank	
	SR	MS	SR	MS	SR	MS	SR	MS	SR	MS
eGARCH-snorm	1.170	1.199	1.391	1.346	1.053	1.045	0.851	0.762	37.500	28.750
eGARCH-std	1.155	1.190	1.368	1.347	0.998	1.041	0.756	0.723	19.500	24.000
eGARCH-sstd	1.130	1.161	1.362	1.413	0.998	1.043	0.796	0.739	21.000	26.500
eGARCH-ged	1.152	1.185	1.345	1.299	**0.998**	1.025	0.757	0.760	17.250	23.000
eGARCH-sged	1.126	1.164	1.341	1.363	0.998	1.025	0.800	0.795	20.000	28.750
gjrGARCH-norm	1.154	1.122	**1.220**	1.322	1.052	1.095	0.811	0.782	23.500	25.500
gjrGARCH-snorm	1.144	1.143	1.264	1.363	1.053	1.092	0.850	0.764	27.750	29.250
gjrGARCH-std	1.139	**1.097**	1.260	1.309	1.009	1.062	0.760	0.727	15.500	14.500
gjrGARCH-sstd	1.120	1.099	1.262	1.352	1.007	1.045	0.800	0.726	16.250	15.500
gjrGARCH-ged	1.136	1.119	1.247	1.595	1.013	1.053	0.761	0.790	15.750	29.000
gjrGARCH-sged	1.124	1.100	1.244	1.289	1.012	1.066	0.802	0.816	17.250	22.500

Note. All values were multiplied by 10^3; Boxes indicate the favored models; bold indicates the second-ranked model.

Table A5. Model confidence set at 10% significance level for 1% VaR forecasts

Model	Oil		Gas		Corn		Soy		Total	
	SR	MS	SR	MS	SR	MS	SR	MS	SR	MS
ARCH-norm	Yes	Yes	Yes	Yes					2	2
ARCH-snorm	Yes	Yes	Yes	Yes					2	2
ARCH-std	Yes			Yes				Yes	1	2
ARCH-sstd	Yes							Yes	1	1
ARCH-ged	Yes			Yes				Yes	1	2
ARCH-sged	Yes			Yes				Yes	1	2
GARCH-norm	Yes	Yes	Yes	Yes					2	2
GARCH-snorm	Yes	Yes	Yes	Yes					2	2
GARCH-std	Yes	Yes	Yes	Yes	Yes				3	2
GARCH-sstd	Yes	Yes	Yes	Yes	Yes	Yes			3	3
GARCH-ged	Yes	Yes	Yes	Yes	Yes				3	2
GARCH-sged	Yes	Yes	Yes	Yes	Yes				3	2
eGARCH-norm			Yes	Yes					1	1
eGARCH-snorm			Yes	Yes					1	1
eGARCH-std	Yes		Yes	Yes	Yes			Yes	3	2
eGARCH-sstd	Yes		Yes	Yes	Yes				3	1
eGARCH-ged	Yes		Yes	Yes	Yes	Yes			3	2
eGARCH-sged	Yes		Yes	Yes	Yes	Yes			3	2
gjrGARCH-norm	Yes	Yes	Yes	Yes					2	2
gjrGARCH-snorm	Yes	Yes	Yes	Yes					2	2
gjrGARCH-std	Yes	Yes	Yes	Yes	Yes			Yes	3	3
gjrGARCH-sstd	Yes	Yes	Yes	Yes	Yes			Yes	3	3
gjrGARCH-ged	Yes	Yes	Yes	Yes	Yes				3	2
gjrGARCH-sged	Yes	Yes	Yes	Yes	Yes				3	2
Total	22	14	20	23	12	3	0	7	54	47

Note. Boxes indicate the favored model, based on the total number of inclusions in the MCS across the four commodities.