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Structural breaks in panel data: COVID-19 pandemic in Russian regions

This paper discusses contemporary methods of testing for structural breaks in panel data. We discuss the approaches for single and multiple breaks testing in cross-sectionally correlated panels. For empirical application, we use weekly data for the COVID-19 pandemic in Russian regions. We identify several structural breaks over the period and add them to the econometric model. The results show a significant difference compared to the model that ignores structural breaks, underscoring the importance of accounting for these breaks when analyzing Russian regional data.

Keywords: structural breaks; models with changing regimes; Russian regions; panel data; COVID-19 pandemic.

JEL classification: C12; C22.

1. Introduction

Structural breaks take a central place in the modern applied econometrics and statistics. Their identification and dating play a key role in the analysis of the dynamics of socio-economic data, changes in financial markets and economic policy. In particular, institutional reforms and external events such as macroeconomic crises, changes in monetary and fiscal policies, technological advances, epidemics, changes in legislation, etc. lead to structural breaks. Ignoring such changes results in model misspecification, distorted understanding of economic mechanisms and misleading policy recommendations.

The advantage of panel data is the ability to simultaneously consider individual heterogeneity and the dynamics of the changes over time. However, due to the dual dimensionality the problem of structural breaks in panels becomes more complex and multifaceted. In contrast, most classical statistical methods developed for time series cannot be directly transferred to the panel context without significant modifications.

In panel models, the identification of structural breaks has particular importance due to the variation of data both over time and across the objects. For instance, during the crisis periods some regions or industries may respond more rapidly (or more significantly) to emerging changes, which leads to an earlier (or more significant) change in the dynamics of economic indicators. In addition, unobserved heterogeneity arising in panel data (e.g., interactive effects, latent factors) complicates

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the ability to distinguish the real breaks from the fluctuations caused by unaccounted individual or time features. In this case, the structural break may be mistaken for the effect of unobserved factors, or vice versa.

Recently, a vast number of new methods were developed to solve the above-mentioned problems of empirical analysis: regularization approaches (group fused Lasso, adaptive Lasso), dynamic programming[2], modified CUSUM-tests and generalizations of Bai and Perron (1998) approach, methods with interactive and random effects. It indicates a high demand for the topic along with the need for further development of theory and practice in analyzing structural changes in panel data.

In this paper, we focus on offline (retrospective) detecting of structural breaks. Nevertheless, there are methods of online detection of breaks going back at least to the 1960s (see, e. g., Shiryaev, 1963; Roberts, 1966). For panel data, however, there are few papers focusing on monitoring of structural breaks. Recently, Horváth et al. (2022) proposed a new CUSUM-type statistic in the cross-sectional dependent panel. See also (Jiang, Kurozumi, 2023) and references therein.

The paper is structured as follows. Section 2 discusses the methods for testing a single structural break. Section 3 considers the methods for testing multiple structural breaks. Section 4 provides an empirical application. The conclusion presents the main findings of the study.

2. Approaches to testing for a single structural break

Without loss of generality, it can be assumed that given a procedure for testing a panel for one structural break, it is not difficult to develop a method for testing the same panel for m_0 structural breaks. Indeed, after the break date identification, we can consider two subpanels (before and after this date) and test each of them separately using the same method. However, two important aspects should be considered. First, the asymptotic properties of almost all tests deteriorate as the panel under consideration decreases. Second, it is necessary to remember about the control of the significance level, since the first type of error accumulates as the testing proceeds. The work of Bai (2010) is fundamental and is devoted to the methods for estimating common structural breaks in the mean and variance in panel data. Unlike the classical problems of determining structural breaks in time series, it becomes possible to obtain consistent estimators of the structural break even for small T and large N in the panel data and to construct reliable confidence intervals based on the asymptotics in N.

Bai (2010) considers the following panel as a basic model:

$$y_{it} = \begin{cases} \mu_{i1} + e_{it}, & t \le k_0, \\ \mu_{i2} + e_{it}, & t > k_0, \end{cases} \tag{1}$$

where $i = 1, \ldots, N$ — the number of time series, $t = 1, \ldots, T$ — the length of each time series, μ_{i1}, μ_{i2} — averages before/after the break, k_0 — unknown date of structural break, e_{it} — stationary errors i. i. d. across i and t.

[2] The Mixed Integer Programming method of estimation the number of breaks is recently developed in (Prokhorov et al., 2025) only for univariate model. The extension to the panel data model is under investigation.

To obtain the date of structural break $\hat{k}$, the means before and after time moment k for each value of k are considered:

$$\bar{y}_{i1}(k) = \frac{1}{k}\sum_{t=1}^{k} y_{it}, \tag{2}$$

and

$$\bar{y}_{i2}(k) = \frac{1}{T-k}\sum_{t=k+1}^{T} y_{it} \tag{3}$$

respectively. For each value of k the sum of squared residuals is calculated for the entire panel:

$$SSR(k) = \sum_{i=1}^{N}\left(\sum_{t=1}^{k}\left(y_{it} - \bar{y}_{i1}(k)\right)^2 + \sum_{t=k+1}^{T}\left(y_{it} - \bar{y}_{i2}(k)\right)^2\right). \tag{4}$$

Then the estimate of the break moment can be obtained based on the minimization of the sum of squared residuals over all possible values of k:

$$\hat{k} = \underset{1\le k\le T-1}{\operatorname{argmin}}\ SSR(k). \tag{5}$$

Bai (2010) demonstrated that the obtained estimator is consistent for k_0 if $N \to \infty$ for fixed T. That implies that equality holds:

$$\lim_{N\to\infty} \mathbb{P}\left(\hat{k} = k_0\right) = 1 \tag{6}$$

under condition that the difference in the panel means does not decrease:

$$\frac{1}{N}\sum_{i=1}^{N}(\mu_{i2} - \mu_{i1})^2 \to \lambda > 0, \qquad \text{for } N \to \infty. \tag{7}$$

The proof is based on the fact that for $N \to \infty$ the deterministic contribution from incorrect panel splitting always exceeds the noise from errors even for very small differences in means (but for $\lambda > 0$). Therefore, $SSR(k)$ is minimized with high probability in the true value of k — and the structural break estimator is consistent. Note that the obtained break date estimator is consistent even if one of the regimes consists of a single observation (the start or end time of the new regime). However, the presence of only one series in the panel $N = 1$ does not allow a consistent estimation of k_0 even for large T.

It was also shown that for small values of the difference in means, a confidence interval of level α is constructed as follows (with $N \to \infty$):

$$\mathbb{P}\left(\left|\hat{k} - k_0\right| \le \alpha_c / A_N\right) \to 1 - \alpha, \tag{8}$$

where

$$A_N = \frac{1}{N}\sum_{i=1}^{N}(\mu_{i2} - \mu_{i1})^2 / \hat{\sigma}_i^2, \tag{9}$$

and α_c is a corresponding percentile of the random variable from formula (10)

$$\underset{l}{\operatorname{argmin}}\left(|l| + 2W(l)\right), \tag{10}$$

where $W(l)$ is a two-side Gaussian random walk: $W(0) = 0$, $W(l) = \sum_{j=1}^{l} \varepsilon_j$ for $l > 0$, $W(l) = -\sum_{j=l+1}^{0} \varepsilon_j$ for $l < 0$, where ε_j, $j = 0, \pm 1, \pm 2, \ldots$ — i. i. d. $N(0,1)$ variables, and l belongs to the set of integers.

Bai (2010) proposes applying the Chow test for each individual time series at the found common structural break date (assuming that the date is exogenous) to determine which time series are subjected to a break. Bai (2010) also investigates the break in the error variance e_{it}. To estimate the break date, the author suggest applying quasi-maximum likelihood (hereinafter QML) methods. The corresponding sum of squared residuals (SSR) is represented as follows:

$$SSR_{QML}(k) = \sum_{i=1}^{N}\left[k \log \hat{\sigma}_{i1}^2(k) + (T-k)\log \hat{\sigma}_{i2}^2(k)\right]. \tag{11}$$

QML gives a consistent estimate of k_0 and is more efficient than conventional OLS, even if there is no break in variance, and for the estimator consistency the relation (12) should be satisfied:

$$\frac{1}{N}\sum_{i=1}^{N}\log\left(\sigma_{i1}^2/\sigma_{i2}^2\right) \to c_\sigma, \tag{12}$$

where c_σ — non-zero constant.

Regarding the multiple structural breaks, Bai (2010) notes that the procedure discussed above can be generalized inline with (Bai, Perron, 1998).

The Monte Carlo simulations also confirm the theoretical conclusions. Even for small values of N, the accuracy of the break dating and the coverage of the confidence intervals are very high, and with increasing N, the intervals narrow and the accuracy of the estimate increases. This method works well for various error distributions ($N(0,1)$, χ_5^2, t_5). In turn, QML works better than OLS in the presence of heteroscedasticity, while OLS is suitable only for cases with break in means.

Baltagi et al. (2025) generalize the approach proposed above to the case of panel data with heterogeneous coefficients in the presence of common unobservable factors (common correlated effects, hereinafter CCE).

The authors consider a following model:

$$y_{it} = x_{it}'\beta_i + e_{it}, \tag{13}$$

where x_{it} is a p-dimensional vector of regressors, e_{it} is the model errors, which have the following form:

$$e_{it} = \gamma_i' f_t + \varepsilon_{it}, \tag{14}$$

f_t is m-dimensional vector of unobserved common factors, γ_i is a factor loading, ε_{it} is a sequence of independent and identically distributed random variables with zero mean and constant variance.

Baltagi et al. (2025) generalize the model (13), assuming the presence of a common structural break for all time series of the panel at one moment in time k_0, that is, the vector β_i has the following form:

$$\beta_i(k_0) = \begin{cases} \beta_i^{(1)}, & t \le k_0, \\ \beta_i^{(2)}, & t > k_0. \end{cases} \quad (15)$$

Note that model (13) can also be generalized by selecting a subset of exogenous regressors that did not sustain structural breaks (the so-called partial structural break model). To eliminate the bias in the coefficient estimates due to the presence of common latent factors f_t that provide cross-sectional dependence, the CCE approach proposed by Pesaran (2006) is used: cross-sectional means for y_{it} and x_{it} are added to the model, which asymptotically approximate the effect of f_t. After that, the moment of the structural break is estimated by minimizing the sums of squared residuals over all possible break dates, similar to (Bai, 2010). The *supF* statistics can be implemented to test for the null hypothesis of no break.

In this case, if β_i is assumed to be a random variable, an estimator of the group means can be consistent with the expectation of β_i when calculated as follows:

$$\hat{\beta}_{gm} = \frac{1}{N} \sum_{i=1}^{N} \hat{\beta}_i(\hat{k}). \quad (16)$$

Baltagi et al. (2016) demonstrated that the obtained estimate $\hat{k}$ is a consistent estimator for k_0 if $N, T \to \infty$. The CCE estimates of the coefficients before and after the break have asymptotically normal distribution similar to the case of known k_0. The resulting technique also extends to the case of multiple breaks using sequential testing algorithms, and also using bootstrapping for the confidence intervals of the structural breaks dates.

The paper tests the proposed method using Monte Carlo simulations. The authors note that the accuracy of the $\hat{k}$ estimate increases rapidly with N and T, even if only a part of the time series exhibits a break. If common unobservable factors are ignored, the accuracy of the estimate is very low, especially with highly correlated errors. This method is also efficient for multiple breaks.

The paper of Baltagi et al. (2017) is devoted to estimating the moment of the structural break in panel data models with different possibilities for the stationarity/non-stationarity of regressors and errors. That implies that the regressors x_{it} and errors u_{it} can be stationary $I(0)$ or non-stationary $I(1)$ processes. In addition, the authors consider the cases when structural break is present and absent. The paper highlights the comparison of the efficiency of the ordinary least squares (OLS) method and the first difference (FD) method as approaches for identifying the moment of the structural break and establishing its asymptotic properties.

In general, the authors consider a panel regression model with a common moment of structural break k_0:

$$y_{it} = \begin{cases} \mu_{1i} + \beta_{1i} x_{it} + u_{it}, & t \le k_0, \\ \mu_{2i} + \beta_{2i} x_{it} + u_{it}, & t > k_0, \end{cases} \quad (17)$$

where x_{it} and u_{it} are assumed as $AR(1)$ processes, which can be stationary ($|\rho| < 1$) or non-stationary ($\rho = 1$):

$$x_{it} = \rho_x x_{it-1} + \eta_{it}, \quad (18)$$

$$u_{it} = \rho_u u_{it-1} + e_{it}. \quad (19)$$

The authors also assume that the magnitude of the break in the coefficients can be small, and it is not required to decrease with increasing dimension.

Baltagi et al. (2017) consider two estimators of the structural break date: $\hat{k}_{OLS}$ and $\hat{k}_{DF}$. The estimate of $\hat{k}_{OLS}$ is obtained similar to Bai and Perron (1998), based on minimizing the sum of squared residuals over all possible break dates. This estimate has a number of advantages. First, for large N ($N \gg T$), the estimator of $\hat{k}_{OLS}$ becomes consistent even with the non-stationarity of x_{it} and u_{it}. Second, a sufficiently high convergence rate was obtained for panel data: if both variables are stationary, then the convergence rate of the estimator is N; for non-stationary processes, it is NT (or N/T if $T/N \to 0$ in the case when the errors are stationary and the regressors are non-stationary). Third, the asymptotic distribution of the $\hat{k}_{OLS}$ estimate remains the same as in (Bai, 2010).

However, the $\hat{k}_{OLS}$ estimator has one significant drawback. If the structural break is actually absent, and x_{it} and u_{it} are both integrated first-order processes, minimizing the sums of squared residuals will erroneously find the structural break in the middle of the sample. Similar conclusions were reached by Bai and Perron (1998) and Nunes et al. (1995) for the one-dimensional case. This phenomenon is called "false break" and is associated with the fact that random walks can be mistaken for a structural break.

Baltagi et al. (2017) propose an alternative estimation method based on constructing first differences (hereinafter FD), which eliminates individual effects and makes the procedure robust to non-stationarity. For all variables in model (17) the first differences were taken:

$$\Delta y_{it} = \begin{cases} \beta_{1i}\Delta x_{it} + \Delta u_{it}, & t \le k_0, \\ \beta_{2i}\Delta x_{it} + \Delta u_{it}, & t > k_0, \end{cases} \tag{20}$$

which results in a model with stationary or nearly stationary variables. The structural break is then estimated in a similar way by minimizing the sums of squared residuals over all possible break dates.

The authors show that the $\hat{k}_{FD}$ estimator is always consistent (at rate N) regardless of the regressor and the error stationarity, even if x_{it} and u_{it} are both $I(1)$. Moreover, this method does not have the problem of spurious breaks. Even if there is no real break, the $\hat{k}_{FD}$ estimator converges to the sample boundaries (beginning or end), unlike $\hat{k}_{OLS}$. The FD estimator is the most robust to model selection, although robustness disappears in case of very strong cross-sectional dependence.

When comparing OLS and FD in Monte Carlo simulations, we can see that for stationary models (with a regressor and an error term $I(0)$) OLS and FD behave similarly. When the regressor or the error term (especially both) are non-stationary, OLS often produces spurious breaks, while FD does not. In examples where there is no break, the FD estimator is always concentrated at the boundaries (0 or T), while OLS prefers the middle of the time interval as the break location. In cases when break exists, the FD estimator converges to the true date faster as N increases. The FD estimator is less sensitive to the break magnitude and the break location, as well as to the error distributions and the choice of N and T.

Antoch et al. (2019) developed a theory for detecting structural breaks in panel data with a short time dimension T and a large number of panels N ($N \to \infty$, T is fixed). Such a scenario is often encountered in finance and economics, where there are many spatial observations (for example, firms, assets and etc.), and the number of observations in time is limited.

The authors propose a general statistical approach for testing the hypothesis of a structural break presence and estimating the break moment, prove the asymptotic properties of test statistics for the case of a short panel, and develop a new bootstrap procedure for obtaining critical values for a fixed T.

A data generation process is represented by the following model:

$$y_{it} = x'_{it}\left(\beta_i + \delta_i \cdot \mathbb{I}\{t \geq k_0\}\right) + e_{it}, \tag{21}$$

where β_i is the parameter before the structural break, δ_i is a break magnitude in the i-th panel after the break date.

Antoch et al. (2019) propose two groups of test statistics to check the presence of a break. First, they propose an analogue of the Wald statistic, which is based on the differences in the coefficient estimates of different modes. This analogue has the following form:

$$U_N(t) = \sum_{i=1}^{N}\left[\left(\hat{\beta}_{i,t} - \hat{\beta}_{i,T}\right)' C_{i,t}\left(\hat{\beta}_{i,t} - \hat{\beta}_{i,T}\right)\right], \tag{22}$$

where $\hat{\beta}_{i,t}$ is OLS-estimate for the first t observations, $\hat{\beta}_{i,T}$ is OLS-estimate for the entire panel, $C_{i,t}$ is a weighting matrix.

The second statistics proposed by Antoch et al. (2019) is CUSUM-statistics, which is calculated based on the sum of squared residuals:

$$V_N(t) = \sum_{i=1}^{N}\sum_{s=1}^{t}\hat{e}_{is}^2, \tag{23}$$

where $\hat{e}_{is}$ is a sample residual of the model estimated over the entire sample:

$$\hat{e}_{is} = y_{is} - x'_{is}\hat{\beta}_{i,T}. \tag{24}$$

If at least for some t the statistics exceeds the critical level, the hypothesis of coefficient stability is rejected. Further reasoning for both types of statistics is the same, so all calculations will be given with the Wald statistics $U_N(t)$.

The procedure for testing for a structural break is as follows. First, for each possible moment t, $U_N(t)$ and/or $V_N(t)$ are calculated. Note that the typical period of minimization/maximization of statistics is the range $d \leq t \leq T-d$ for a small d, so that both subsamples are present. Next, the statistics are standardized for correct comparison. The statistics are normalized to the estimate of their asymptotic variance:

$$U_N^*(t) = N^{-1/2}\left(U_N(t) - A_N^{(1)}(t)\right), \tag{25}$$

where $A_N^{(1)}(t)$ is a normalizing function, which is calculated based on σ_i^2 estimates.

Then the resulting value of the test statistic is calculated as follows:

$$u_N = \max_t\left|U_N^*(t)\right|. \tag{26}$$

Antoch et al. (2019) show that the statistics represented by formula (25) have asymptotically multivariate normal distribution:

$$N^{-1/2}\left(U_N(t) - A_N^{(1)}(t)\right) \underset{d}{\rightarrow} \mathcal{N}\left(0, \Gamma(t, t')\right), \tag{27}$$

where Γ is some covariance matrix whose parameters can be estimated from the data.

To obtain critical values, a wild bootstrap procedure was proposed.

1. For each $i = 1, \ldots, N$ and $t = d, \ldots, T - d$, calculate the values:

$$\varphi_{i,t} = \left(\hat{\beta}_{i,t} - \hat{\beta}_{i,T}\right)' C_{i,t}\left(\hat{\beta}_{i,t} - \hat{\beta}_{i,T}\right) - \frac{1}{N}\sum_{j=1}^{N}\left(\hat{\beta}_{i,t} - \hat{\beta}_{i,T}\right)' C_{j,t}\left(\hat{\beta}_{j,t} - \hat{\beta}_{j,T}\right).$$

2. Generate independent random variables $\zeta_i \sim N(0,1)$.
3. Generate bootstrap observations as follows: $\varphi^*_{i,t} = \zeta_i \varphi_{i,t}$.
4. Obtain a bootstrap process: $U^*_N(t) = N^{-1/2}\sum_{i=1}^{N}\varphi^*_{i,t} = N^{-1/2}\sum_{i=1}^{N}\zeta_i\varphi_{i,t}$.
5. Obtain a bootstrap version of u^*_N, by maximizing $U^*_N(t)$: $u^*_N = \max_{t\in[d,T-d]}\left|U^*_N(t)\right|$.
6. Repeat the steps 2–5 B times to obtain the empirical distribution of the statistics u^*_N.
7. For α-level test the bootstrap critical level is $(1-\alpha)\times 100\%$-quantile by $u_N^{*,(b)}$, $b = 1, \ldots, B$.
8. Reject H_0 (absence of the structural break), if the empirical $u_N > c^*_{N,\alpha}$.

Simulations demonstrate that the test size is close to the nominal one even for small T (from 50 observations) if bootstrapping or correct asymptotic normalization is used. The power of the test increases rapidly with N even for a moderately large panel (100–500 or more). The test works well even if only some of the series are sustained by a structural break. It is important to remember that if the correlation within the panel is high, the behavior of the statistics changes and requires separate analysis. Antoch et al. (2019) also show that the proposed statistics can be adjusted for different types of breaks: for a break in the coefficients, a break in the mean, a break in the variance, etc. using the type of weight matrix $C_{i,t}$.

Zhu et al. (2020) continue to develop a test for identifying the date of a structural break for large N and small T. The authors focus on the case when there are common factors in the panel. In prior studies the presence of common factors obliged the researches to use the long panels with large T. Zhu et al. (2020) allow for small panels and consider the following model:

$$y_{it} = \begin{cases} x'_{it}\beta^{(1)} + \lambda'_i f_t + \varepsilon_{it}, & t = 1, \ldots, \tau - 1, \\ x'_{it}\beta^{(2)} + \lambda'_i f_t + \varepsilon_{it}, & t = \tau, \ldots, T, \end{cases} \tag{28}$$

where f_t represents common factors (their number r can be unknown). The model allows for a break in any component of β, and also allows for breaks in the coefficients of the common factors λ_i.

The main statistical approach used by the authors is the generalized method of moments (hereinafter GMM). In this case, moment conditions are the following:

$$\mu_{\tau,i}(\theta_\tau) = Z_i\left(y_i - X_i^{(1)}\beta_1 - X_i^{(\tau)}\beta_2\right) - S\left(I_T \otimes G\right) f, \tag{29}$$

where Z_i is a matrix of instruments, $G = \mathrm{E}\left(w_i \lambda_i'\right)$, S is a diagonal matrix, f is a set of factors on the whole t, θ_τ is a vector consisting of the vectors, which is the vertically decomposed matrix G, f, β_1 and β_2.

The GMM estimator can be obtained by minimizing the quadratic form $Q_\tau\left(\theta_\tau\right)$:

$$Q_\tau\left(\theta_\tau\right) = \mu_\tau(\theta_\tau)' W \mu_\tau\left(\theta_\tau\right), \tag{30}$$

where W is a positive definite matrix of weights and $\mu_\tau(\theta_\tau) = \mathrm{E}\,\mu_{\tau i}(\theta_\tau)$. The resulting GMM estimator is represented by equation (31):

$$\hat{\theta}_\tau = \operatorname*{argmin}_{\theta_\tau} Q_\tau\left(\theta_\tau\right). \tag{31}$$

To test the null hypothesis $\mathrm{H}_0 : \beta_1 = \beta_2$ (absence of the structural break) against alternative $\mathrm{H}_1 : \beta_2 \neq \beta_1$ (presence of the structural break) the distance-based statistics and LM statistics are calculated. The distance-based statistics for a given break moment τ has the following form:

$$D_\tau = N\left[Q_1\left(\hat{\theta}_1\right) - Q_\tau\left(\hat{\theta}_\tau\right)\right]. \tag{32}$$

If the moment of the structural break is unknown, the authors propose to take the maximum over all τ:

$$D_{\max} = \max_{\tau \in \{\tau_1, \ldots, \tau_L\}} D_\tau, \tag{33}$$

where L is the number of points, in which there may presumably be a structural break.

The LM-statistics is calculated as follows:

$$LM_\tau = N A_\tau \hat{U}^{-1} A_\tau, \tag{34}$$

where A_τ is an estimated gradient of the moment function, $\hat{U}$ is the estimator of the covariance matrix.

If $N \to \infty$ and fixed T statistics D_τ and LM_τ have asymptotic distribution χ^2 with the number of the degrees of freedom equal to dimensions of changing coefficients:

$$D_\tau \xrightarrow[d]{} \chi_k^2, \tag{35}$$

where k is the number of coefficients, which are exposed by a structural break. This limiting distribution opens the possibility for us to use the wild bootstrap to obtain critical values of the $D_{\max}$ statistics.

The comparative analysis conducted using Monte Carlo simulations demonstrated that the distance-based test controls the size better and keeps it within the required limits even for small T, 6–9. The LM test overestimates the size for small N, and becomes correct for $N \geq 300$. At the same time, both tests perform well with large changes in the coefficients even for small T, the power quickly approaches 1 with increasing N. The frequency of correct localization of the break date increases with N and with the increase in the break magnitude. If the number of common factors is unknown, the distance-based test is more robust; the test based on LM statistics exhibit lower power.

Karavias et al. (2023) summarize several prior studies and develop a new toolkit for identifying and analyzing common structural breaks in panel data with interactive effects, i.e. with an unspecified common component that is highly correlated with the regressors. The authors motivation lies in the need of analyzing the COVID-19 consequences for global stock markets. The pandemic caused a significant structural break that affected many countries and markets, i.e. the reaction of stock markets turned out to be nonlinear and time-dependent. Most existing methods either assume a large time dimension T or do not consider highly correlated errors between the panels.

As a basic model, the authors consider a panel regression with a possible common structural break and interactive effects (unobserved common regressors):

$$y_{it} = x_{it}'\beta + z_{it}'\delta\mathbb{I}\left\{t > k_0\right\} + e_{it}, \tag{36}$$

where z_{it} is a subset of regressors whose coefficients can change at time k_0, β, δ are the vectors of coefficients before the break and the break magnitude for the variables sustained by a structural break, respectively, e_{it} is the error term, containing unobserved common factors, see formula (14).

To estimate the moment of the break $\hat{k}_0$, the procedure of minimizing the sums of squared residuals as in (Baltagi et al., 2016) is used. However, before the moment estimation the authors add cross-sectional averages to eliminate the effects of common factors (CCE).

Testing for a structural break is carried out using the generalized Wald statistic, which is presented as follows:

$$W\left(k_0\right) = \left[\hat{\delta}\left(k_0\right)\right]' \widehat{Cov}^{-1}\left[\hat{\delta}\left(k_0\right)\right]\hat{\delta}(b), \tag{37}$$

where $\hat{\delta}\left(k_0\right)$ is the estimate of the structural break magnitude for the date k_0, $\widehat{Cov}$ is the estimated covariance matrix of the corresponding coefficients.

If the date of the break is unknown, SW-statistics is calculated, which is the supremum of $W\left(k_0\right)$ over all k from the admissible set $\mathcal{B}$ (as a rule, the first and the last 15% of the observations are not included). This statistic has the following form:

$$SW = \sup_{k\in\mathcal{B}} W\left(k\right). \tag{38}$$

Karavias et al. (2023) note that the asymptotic distribution of SW-statistics corresponds to a Bessel process of order r (where r is the number of breaked regressors). If $r \geq 2$, then the Bessel process of order r is defined using the equality (39):

$$X_t = \left\|W_t\right\|, \tag{39}$$

where W_t is an r-dimensional Brownian motion, and $\|\cdot\|$ is the Euclidean norm in the space $\mathbb{R}^r$. The critical values for this statistic are given in (Andrews, 1993). Coincidently, Karavias et al. (2023) prove that the obtained statistical procedure and the structural break estimator are not only robust to common factors and small values of T, but they are also robust to high correlation between regressors.

3. Methods for multiple structural breaks testing

Qian and Su (2016a) extended the approach of (Qian, Su, 2016b) for the case of general structural breaks in panel models based on the Adaptive Group Fused Lasso (hereinafter AGFL) regularization method. A model with fixed effects is considered:

$$y_{it} = x'_{it}\beta_t + c_i + u_{it}, \tag{40}$$

where c_i are fixed effects, x_{it} are regressors, u_{it} are errors with zero mean.

Coefficients β_t have the following representation:

$$\beta_t = \begin{cases} \alpha_1, & 1 \le t < T_1, \\ \alpha_2, & T_1 \le t < T_2, \\ \dots \\ \alpha_{m+1}, & T_m \le t \le T, \end{cases} \tag{41}$$

the number of structural breaks is m and the break dates T_j are unknown.

To eliminate fixed effects, we consider the following first differences regression:

$$\Delta y_{it} = x'_{it}\Delta\beta_t + \Delta u_{it}. \tag{42}$$

The basic idea of the AGFL method is to minimize the sum of squared residuals with a penalty for large differences in the coefficients of adjacent periods:

$$\min_{\{\beta_t\}} \left\{ \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=2}^{T} (\Delta y_{it} - x'_{it}\Delta\beta_t)^2 + \lambda \sum_{t=2}^{T} w_t \left\| \beta_t - \beta_{t-1} \right\| \right\}, \tag{43}$$

where w_t are adaptive weights, λ is a regularization parameter, $\| \cdot \|$ is an Euclidean norm.

The AGFL estimator of the coefficients β_t is

$$\left\{ \hat{\beta}_t \right\} = \operatorname*{argmin}_{\{\beta_t\}} \left\{ \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=2}^{T} (\Delta y_{it} - x'_{it}\Delta\beta_t)^2 + \lambda \sum_{t=2}^{T} w_t \left\| \beta_t - \beta_{t-1} \right\| \right\}, \tag{44}$$

where λ is selected based on the minimization of the information criterion

$$IC(\lambda) = \log\hat{\sigma}^2(\lambda) + (m+1)\,p\,\frac{\log NT}{NT}, \tag{45}$$

where $\hat{\sigma}^2$ is estimated variance of the errors.

In the case of endogeneity, Penalized GMM (hereinafter PGMM) is used, where the objective function is constructed based on moment conditions with instruments, and the penalty and subsequent AGFL estimation are similar.

For the set of the regimes obtained in the previous step, a classical non-regularized (post-Lasso) regression is used in each regime to estimate β_j and standard errors more accurately.

The key difference between AGFL and the tests from the previous section is that the test for the presence of a break is implemented as a part of the estimation procedure: if the optimal value of λ according to IC gives all β_t equal to each other (no breaks), then there is no reason to reject H_0. If at least 1 break is found, the test rejects H_0.

Qian and Su (2016a) demonstrate that the proposed estimators of the breaks magnitudes are consistent and have a multivariate normal asymptotic distribution, while the convergence rate is $\sqrt{N}$. This algorithm works well on Monte Carlo simulations even in the presence of endogeneity and for relatively small N (from 50) and T (from 6).

Li et al. (2016) show that the AGFL method can be used to determine multiple breaks in the case where the original model contains common factors f_t (or interactive fixed effects). The authors demonstrate that the AGFL estimator has the same properties as for the case of standard fixed effects.

Boldea et al. (2020) emphasized that most standard approaches require removing individual fixed effects by centering or taking first differences, which has a negative effect on efficiency and often leads to a loss of information, especially in short panels when T is not very large. Boldea et al. (2020) proposed a new approach: break dates in the panel can be found directly on the original data using OLS, without removing individual effects. Even though unsmoothed individual effects make the coefficient estimates inconsistent, the break dates themselves are found correctly.

The authors consider a panel regression with fixed effects as

$$y_{it} = x_{it}'\beta_j + c_{it} + \varepsilon_{it}, \tag{46}$$

where $t \in I_j = [T_{j-1}+1, T_j]$, c_{it} are individual fixed effects allowing their change over time.

Let m_0 be the number of the real breaks at unknown time points $T_1^0, ..., T_{m_0}^0$. Then, in order to obtain their estimates, we assume that there are m breaks, which occur at time points $\{T_j\}$, $j = 1, \ldots, m$. We divide the panel into $m+1$ segments and estimate the combined model using OLS for vectorized data with a block-diagonal matrix of regressors X. Then, we can calculate the sum of squared residuals as

$$SSR = S_{NT}(\beta) = \frac{1}{NT}\|y - X\beta\|^2. \tag{47}$$

After that, we minimize the information criterion

$$IC(m) = \log S_{NT}\left(\hat{\beta}_{m,\hat{\lambda}_m}\right) + \left[3m + (m+1)p\right]\frac{\operatorname{loglog}(NT)}{NT}, \tag{48}$$

over all values of m and $\{T_j\}$, where p is the number of the regressors.

It turns out that for large N and fixed T the number and timing of breaks are estimated consistently, even if the individual effects are uncontrolled noise that depends on time. However, this procedure does not answer the question of where the break occurs: in the coefficients or in the individual effects. Indeed, since OLS does not identify β itself due to the bias, a jump in the "combined" value is found:

$$\gamma_j^0 = \beta_j^0 + Q_j^{-1}a_j^0, \tag{49}$$

where Q_j^0 is a limiting covariance matrix of the regressors of segment j, a_j^0 is a covariance of the regressor with the individual effect within the segment. Therefore, the bias can be either in β, or in the covariance relationship a_j^0 (endogeneity bias), or in both parameters.

Boldea et al. (2020) propose a separate procedure to determine whether the detected break is a break in the slope β, a break due to endogeneity a_j^0, or a combination of both. First, it is necessary to determine the number and dating of structural breaks, minimizing the information criterion (48). Within each segment, the FE model is estimated. After that, a covariance bias matrix is constructed:

$$\hat{a}_j = \frac{1}{N\left|I_j\right|}\sum_{i,t\in I_j} x_{it}\left(y_{it} - x_{it}'\hat{\beta}_j\right), \tag{50}$$

where $I_j = \left[T_{j-1}+1, T_j\right]$.

At the structural break points of interest, a two-side Chow test is used for the null hypothesis that $\mathrm{H}_0 : \beta_j = \beta_{j+1}$ and for the hypothesis that $\mathrm{H}_0 : a_j = a_{j+1}$, which are tested separately.

The test statistic is an analogue of the Chow test statistic for panel data:

$$W = \left(\hat{\theta}_1 - \hat{\theta}_2\right)'\left[\widehat{\mathrm{Var}}\left(\hat{\theta}_1\right) + \widehat{\mathrm{Var}}\left(\hat{\theta}_2\right)\right]^{-1}\left(\hat{\theta}_1 - \hat{\theta}_2\right) \sim \chi_p^2, \tag{51}$$

where $\hat{\theta}_j$ is a coefficient or a covariance bias. Since the tests are performed simultaneously, a Bonferroni correction should be performed. If the break is significant only for β, it is a change in the coefficient; if only for a, it is a change in the covariate bias (e.g., endogeneity has changed); if both, it is a complex change.

Note that this method has a significant advantage over the FD-approach discussed earlier. If the data contains regressors that do not change over time (e.g., a person's gender), taking the first difference will remove this regressor from the model, unlike the approach in (Boldea et al., 2020).

The Monte Carlo results demonstrate that the proposed method performs much better than the FD-approach for $T \leq 20$, especially in terms of the accuracy of the estimated number of breaks and their locations. For small changes in the coefficients, break detection works worse, but the power is still adequate for reasonable changes. False breaks are very rare (false positive rate FPR < 1%). All of this is a consequence of the refusal to remove individual effects before searching for breaks, preserving the maximum information content of the data, which is critical for short panels.

Kaddoura and Westerlund (2023) continue to develop methods for detecting structural breaks in panel data in a situation where T is fixed and small, and N is large enough. The authors develop a test that is robust to hidden (random) interactive effects. The proposed approach is applicable even with fixed T and arbitrarily complex heterogeneity across N units, which makes it unique compared to existing methods.

The panel regression model is considered:

$$y_{it} = x_{it}'\beta_t + \lambda_i' f_t + \varepsilon_{it}, \tag{52}$$

where $\lambda_i' f_t$ is a random interactive effect (hidden factors or shocks specific to i and changing over time t). It is assumed that β_t takes $m+1$ different values $\alpha_1, \ldots, \alpha_{m+1}$, separated by unknown break dates $T_1 < \cdots < T_m$.

To simultaneously determine the number and localization of breaks, Kaddoura and Westerlund (2023) propose a modification of the group fused lasso method (GFL), called post-demeaned Lasso least squares (hereafter PDL2S). The procedure includes two stages. First, the influence of interactive effects is eliminated from the model by centering (subtracting the average values for i).

Then, panel data adapted GFL is applied. Estimator of β_t is a solution of the following optimization problem:

$$\min_{\beta_t}\left\{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it}-x_{it}'\beta_t)^2+\gamma\sum_{t=2}^{T}w_t\left\|\beta_t-\beta_{t-1}\right\|\right\}, \tag{53}$$

where w_t are specific data-dependent weights, $\gamma > 0$ is a penalty parameter. Next, based on the obtained partition of the time interval, the coefficient estimates are calculated using the least squares method, and the mean squared forecast error $\hat{\sigma}^2(\gamma)$ is estimated.

The parameter γ is selected from the minimizing the information criterion:

$$IC(\gamma)=\log\hat{\sigma}^2(\gamma)+\varphi\,\frac{\left[m(\gamma)+1\right]p}{NT}, \tag{54}$$

where $m(\gamma)$ is the number of breaks for a given γ, p is the number of the regressors, $\varphi=\log(NT)$.

The authors demonstrate that the estimators of the number of breaks m and their dates T_j are consistent even for fixed T. The estimators of α_j are consistent and asymptotically normal with rate $\sqrt{N}$. Arbitrary random interactive effects are admissible. Among the limitations, it is necessary to note the need for independence of individual shocks ε_{it} and "weak" exogeneity of the regressors x_{it}. The method is especially effective for large N and a fixed, relatively small T.

The procedure for testing for the presence of a structural break includes two steps: first, the PDL2S method is applied, then the Wald test between neighboring regimes or the global test for no change is carried out. The null hypothesis is formulated as follows:

$$\mathrm{H}_0:\alpha_1=\alpha_2=\cdots=\alpha_{m+1}. \tag{55}$$

The Wald-test statistic between the two neighboring regimes is calculated as:

$$W=(\hat{\alpha}_1-\hat{\alpha}_2)'\left[\mathrm{Var}(\hat{\alpha}_1)+\mathrm{Var}(\hat{\alpha}_2)\right]^{-1}(\hat{\alpha}_1-\hat{\alpha}_2), \tag{56}$$

and for $N\to\infty$ has asymptotic χ^2-distribution.

Therefore, the proposed approach allows for fully automating the process of identifying and localizing structural breaks, distinguishing between the absence of breaks, single or multiple breaks.

The Monte Carlo simulation results demonstrate the high accuracy of this approach: it consistently estimates the true number of breaks even for small values of T (for example, $T=5,10,20$) and sample sizes N from 25 to 200. The method demonstrates good robustness for both stationary and non-stationary hidden factors f_t and is characterized by a low false positive rate.

Ditzen et al. (2025b) propose a new two-step procedure for identifying multiple breaks in a model with interactive fixed effects, combining Pesaran's CCE estimator and a generalization of the Bai and Perron (1998) algorithm to the case of panel data. The authors consider the following panel regression:

$$y_{it}=x_{it}'\beta_j+z_{it}'\theta+\lambda_i'f_t+\varepsilon_{it}, \tag{57}$$

where $t\in I_j=\left[T_{j-1}+1,T_j\right]$, $j=1,\ldots,m$, $T_1,\ldots,T_m$ are the breaks dates, $T_0=1$, $T_{m+1}=T$, x_{it} is the vector of the regressors, which allows for structural breaks in the coefficients; z_{it} are control variables, in which structural breaks are not allowed; $\lambda_i' f_t$ are scalar interactive effects reflecting common shocks in the dynamics of y, acting with different strengths for each i. The number of structural breaks is unknown. The testing procedure consists of two steps. In the first step, to eliminate the influence of f_t, the average values by i of all regressors (cross-sectional averages) are included in the regression — by analogy with (Pesaran, 2006). The resulting model is:

$$\tilde{y}_{it} = \tilde{x}_{it}' \beta_j + \tilde{z}_{it}' \theta + \tilde{\varepsilon}_{it}, \tag{58}$$

where the variables now are the projection of the original values on the cross-sectional averages.

Further Ditzen et al. (2025b) propose a panel generalization of the Bai and Perron (1998) methodology. Instead of a one-dimensional time series, the search for breaks is carried out on a multidimensional panel. All key Bai-Perron procedures — the *supF*/Wald test, the *WDmax* and *UDmax* tests, dynamic programming for minimizing the sum of squared residuals, estimation and calculation of asymptotic confidence intervals are adapted to the panel case. The break date estimators have a convergence rate N for large N and small T, and the confidence intervals for these dates are the same as in (Bai, Perron, 1998).

To test the null hypothesis of no break against the alternative that there are m breaks, where m is specified by the researcher, Ditzen et al. (2025b) propose the following *supF* statistic:

$$supF(m) = \sup_{\tau_m} F(\tau_m), \tag{59}$$

where $F(\tau_m)$ is an observed value of the F-statistics when testing the hypothesis for the absence of the breaks against the alternative of m breaks at break date vector τ_m. Note that breaks cannot occur in the first and last $(100 \cdot \varepsilon)\%$ of observations, and that the minimum number of observations between breaks must be equal to εT.

If the null hypothesis implies that there are no breaks, and the alternative hypothesis is that there are breaks from 1 to m, then Ditzen et al. (2025b) propose using *WDmax* statistics:

$$WDmax(m_{\max}) = \max_{1 \le s \le m_{\max}} \frac{c_{\alpha,1}}{c_{\alpha,s}} supF(s), \tag{60}$$

where $c_{\alpha,s}$ is a critical value of $supF(s)$-statistics at the significance level a and s breaks. If $c_{\alpha,s} = c_{\alpha,1}$, the $WDmax(m_{\max})$ statistics reduces to $UDmax(m_{\max})$.

If the null hypothesis implies that there are s breaks, and the alternative is that the number of breaks equals to $s + 1$, then Ditzen et al. (2025b) suggest using the conditional F-statistics:

$$F(s+1 \mid s) = \sup_{1 \le j \le s} \sup_{\tau \in \hat{\boldsymbol{\tau}}_j} F(\tau \mid \hat{\boldsymbol{\tau}}_{\mathbf{s}}), \tag{61}$$

where $\hat{\boldsymbol{\tau}}_{\mathbf{s}}$ are the break dates estimates of s structural breaks under the null hypothesis, and the new break $s+1$ should not be too close to any of the breaks already identified. This procedure is designed to test for additional break sequentially for $s = 0, 1, \ldots$ until the corresponding null hypothesis is rejected.

Ditzen et al. (2025b) note that a similar model is considered in (Baltagi et al., 2016), but with a single structural break. Under stationarity conditions, the model considered in (Baltagi et al., 2017) is also similar to the one studied in (Ditzen et al., 2025b). However, the previous models do not include interactive effects and also assume only one break.

Baltagi et al. (2025) extend the approach proposed by Bai and Perron (1998) to the case of multiple structural breaks in the coefficients in non-stationary heterogeneous panels with common factors. The authors demonstrate that the use of the CCE method in combination with the SSR minimization procedure yields consistent estimates of break dates for $T, N \to \infty$, and if β_i are assumed to be random variables, then the estimator of the group means $\hat{\beta}_{gm}$ has an asymptotically normal distribution, while the estimators of $\hat{\beta}_i$ themselves are also normal.

It is also possible to develop the methods for detecting the structural breaks based on Information criteria. Ninomiya (2005) for univariate Gaussian change-point model and Kurozumi, Tuvaandorj (2011) for multivariate model investigated the penalty term in constructing of Information criteria. It turns out that we need to penalize a break 3 times more than parameters (the penalty should contain $3m+p$, not $m+p$, where m is number of breaks and p is the number of parameters which depends on m). The reason is that the number of estimated break points and the number of regression parameters affect the asymptotic expectation of the residual sum of squares differently (see (Hall et al., 2017)). The extensions of Information criteria for break detecting to the panel data context are needed but are beyond the scope of the present paper.

4. Empirical application

To demonstrate the importance of accounting for structural breaks in panel data analysis, we use a sample of weekly coronavirus (COVID-19) cases in Russian regions. We might anticipate the reader that the data for the empirical analysis are highly criticized for its infidelity and the obtained results are quite evident. Nevertheless, the data is suitable for demonstration of the importance of structural breaks identification and accounting. First, we use the data for the period from April 27 to December 31, 2020 excluding the later period due to the start of the vaccination. Additionally, we extend the dataset till May 14, 2023 for the robustness check. The dependent variable is the COVID-19 mortality rate[3]. As the explanatory variables we implement the infection rate and the average weekly temperature[4] in the region (as a control factor). The model can be generally represented by the following equation:

$$death_{i,t} = \beta_j \,\Delta infected_{i,t-1} + \delta_j \, temperature_{i,t} + \mu_{j,i} + \gamma_i CSA_t + \varepsilon_{i,t}, \quad j = 1,\ldots,m+1, \qquad (62)$$

where $death_{i,t}$ is the number of deaths in week t for region i, $\Delta infected_{i,t-1}$ is the first difference of the amount of infected people in the week $t-1$ for region i, $temperature_{i,t}$ is the average temperature in the week t for region i, CSA_i are the cross-sectional averages of the regressor $infected_{i,t-1}$, which equals to $N^{-1}\sum_{i=1}^{N} infected_{i,t-1}$, $\mu_{j,i}$ are the regional fixed effects, which are assumed to be sustained by the structural break[5,6].

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[3] https://russian-trade.com/coronavirus-russia/.

[4] https://www.gismeteo.ru/diary/4720/2020/3/.

[5] We use the set of commands *xtbreak*, developed by Ditzen (2018) for Stata.

[6] The prior analysis on (non) stationarity of the panel data demonstrates the non-stationarity of the number of infected people and stationarity of the number of deaths and temperature.

At the initial stage of the analysis, we estimate a simplified version of the model without control factors. In the second model we add the temperature as a control factor. Each model is estimated using two approaches: the method for detecting multiple structural breaks in panel data with interactive fixed effects proposed by Ditzen et al. (2025b), and the method for estimating heterogeneous coefficients using common correlated effects in a dynamic panel with a large number of observations by groups and periods (Ditzen, 2018). The approach proposed by Ditzen et al. (2025b) allows us to identify structural breaks and estimate regressions with regime changes. The second approach is designed to estimate panel data based on cross-sectional regression and group mean estimation, which considers fixed effects and cross-sectional correlation effects.

Table 1 represents the results of the *UDmax* test, which tests the null hypothesis of the structural breaks absence against the alternative hypothesis, which assumes the presence of structural breaks. According to the test statistics, the null hypothesis is rejected, which indicates the presence of structural breaks in the model. The sequential tests in Table 2, on the contrary, do not detect structural breaks, because the hypothesis of no breaks is not rejected in favor of the hypothesis of only one break. Due to the inconsistency of the results, Table 3 presents the result of testing the hypothesis of no breaks against the alternative of two breaks, and this hypothesis is rejected. In turn, the hypothesis of two breaks is not rejected (according to Table 2), which leads to the choice of the model with two breaks.

Table 1. The results of the $UDmax$ test — H_0 : no breaks vs. H_1 : $1 \le s \le 5$ breaks

		Bai–Perron critical values		
	Test statistic	1% critical value	5% critical value	10% critical value
UDmax	14.95	12.37	8.88	7.46

Table 2. Sequential test for multiple breaks with unknown dates

		Bai–Perron critical values		
	Test statistic	1% critical value	5% critical value	10% critical value
$F(1\|0)$	3.57	12.29	8.58	7.04
$F(2\|1)$	0.44	13.89	10.13	8.51
$F(3\|2)$	0.56	14.80	11.14	9.41
$F(4\|3)$	6.76	15.28	11.83	10.04
$F(5\|4)$	5.49	15.76	12.25	10.58

Table 3. The results of the $supF(2|0)$ test — H_0 : no breaks vs. H_1 : 2 breaks

		Bai–Perron Critical Values		
	Test Statistic	1% critical value	5% critical value	10% critical value
$supF(2\|0)$	109.24	9.36	7.22	6.28

Table 4 represents the estimated dates of the breaks with confidence intervals. Week 11 corresponds to the period from July 5 to July 12, 2020, and week 27 corresponds to the period from October 25 to November 1, 2020. The identified structural breaks generally correspond to the change in the dynamics of COVID-19 mortality, which characterizes the onset of two waves of the 2020 pandemic — summer and winter (Karpova et al., 2022).

Table 4. The estimation of the structural break dates

Date	95% significance interval	
Week 11	Week 10	Week 12
Week 27	Week 26	Week 28

The estimation results of the three regimes, which are distinguished based on the identified structural breaks, are provided in Table 5. The results demonstrate that the number of infected people has a significant impact on the increase in mortality in Russian regions, but the magnitude of this effect varies depending on the period. The first and third regimes correspond to the first (summer) wave of the pandemic and the beginning of the second (winter) wave of the pandemic. The lowest coefficient in the case of the second regime indicates a decrease in mortality from COVID-19, which corresponds to the period of declining infection rate and an attempt to adapt the healthcare system and government regulation in the field of rapid response to emerging external shocks. It is worth noting that the increase in infected people had a more significant impact on the increase in mortality from COVID-19 in the case of the third regime, which is explained by the observed peak in mortality from COVID-19 in December 2020.

Table 5. Estimation of the regimes

	Regime 1	Regime 2	Regime 3
Deaths	4.893**	3.304*	8.170*
	(2.393)	(1.749)	(5.083)
Number of observations 2890			

Note. Standard errors are in parenthesis: *** — $p < 0.01$, ** — $p < 0.05$, * — $p < 0.1$.

Table 6 provides the estimated coefficients of the variables in each individual regime, which is a detailed version of Table 5. In the case of the first regime, the increase in the number of infected people has no significant impact on mortality, while the mortality rate is significant at the 5% significance level, which can be explained by the relatively low increase in the number of infected in the first wave of the pandemic in Russian regions. As expected, in the second and third regimes, there is a significant positive impact of the increase in the number of infected on mortality caused by coronavirus. Coincidently, the coefficient in the third regime has the highest value, which confirms the burst of morbidity and allows us to identify a new wave of the pandemic.

Table 6. The effects of the regressors impact accounting for the structural breaks

	β_1	β_2	β_3	μ_1	μ_2	μ_3
Deaths	–0.0137	0.0428***	0.0681***	4.907**	3.261*	8.101
	(0.0215)	(0.0081)	(0.0169)	(2.387)	(1.748)	(5.087)
R^2	0.774					

Note. Standard errors are in parenthesis: *** — $p < 0.01$, ** — $p < 0.05$, * — $p < 0.1$. Regional fixed effects and cross-sectional averages are included in the model. The number of observations is 2890, the number of groups is 85.

When estimating the initial model, the consideration of structural breaks in the coronavirus infection rate and deaths cases dynamics allows us obtaining robust estimates (which is further confirmed by the model with a control variable) and determine the degree of the infection rate impact on the mortality increase. To compare the results, we provided the estimates of the initial model specification without structural breaks in Table 7. First, ignoring structural breaks does not allow us to obtain robust results when estimating the model using different methods, despite controlling for individual fixed effects and cross-sectional averages. Second, depending on the estimation approach, we obtain false significance of the regression coefficients, which can lead researchers to incorrect conclusions about the impact of the increase in the number of infected people on the mortality rate. In turn, to confirm our conclusions, we estimate a model with a control factor.

Table 7. The results of the model without structural breaks

	POLS	Panel CCE	Pooled mean group
β	0.00425	–0.00248***	–0.0209
	(0.00489)	(0.000544)	(0.0354)
μ	276.5***	0.343	—
	(30.38)	(5.254)	
R^2	0.715		0.921

Note. Standard errors are in parenthesis: *** — $p < 0.01$, ** — $p < 0.05$, * — $p < 0.1$. Regional fixed effects and cross-sectional averages are included in the model. The number of observations is 2890, the number of groups is 85.

Adding the average weekly temperature in the region to the model also reveals the presence of structural breaks according to the results of the *UDmax* test (Table 8). However, the results of the sequential test for multiple breaks with unknown dates allow us to identify three structural breaks (Table 9), which fall on the periods from July 5 to 12, 2020, from October 11 to 18, 2020, and from November 15 to 22, 2020 (Table 10).

Table 8. The results of the *UDmax* test — H_0 : no breaks vs. $\mathrm{H}_1 : 1 \le s \le 5$ breaks

		Bai–Perron critical values		
	Test Statistic	1% critical value	5% critical value	10% critical value
UDmax	15.38	7.71	5.85	5.08

Table 9. Sequential test for multiple breaks with unknown dates

		Bai–Perron critical values		
	Test statistic	1% critical value	5% critical value	10% critical value
$F(1\|0)$	6.45	7.68	5.74	4.91
$F(2\|1)$	10.98	8.42	6.47	5.70
$F(3\|2)$	10.98	8.86	7.01	6.14
$F(4\|3)$	3.90	9.34	7.42	6.45
$F(5\|4)$	1.71	9.59	7.64	6.74
The number of the breaks			3	

Table 10. The estimation of the structural breaks dates

Date	95% significance interval	
Week 11	Week 10	Week 12
Week 25	Week 24	Week 26
Week 30	Week 29	Week 31

In the case of a model with a control variable, we distinguish four regimes. Table 11 represents the estimation results of the four regimes on mortality. According to the results of empirical modelling, the first and second regimes generally do not demonstrate a statistically significant impact on mortality, while the third and fourth regimes are significant at 5 and 1% significance levels, respectively. The highest coefficient for the regime is observed in the case of the fourth period, which characterizes the second wave of the spread of coronavirus infection in the regions of Russia.

Table 11. Estimation of the regimes

	Regime 1	Regime 2	Regime 3	Regime 4
Deaths	4.893**	3.304*	8.170*	21.32***
	(2.393)	(1.749)	(5.083)	(3.775)
Number of observations		2890		

Note. Standard errors are in parenthesis: *** — $p < 0.01$, ** — $p < 0.05$, * — $p < 0.1$.

The impact of the regimes (Table 12) demonstrates a positive significant impact of the increase in the number of people infected with COVID-19 in the case of the third and fourth regimes, which confirms the previously obtained results. Coincidently, the average weekly temperature indicator leads to an increase in mortality, since in addition to the peak of coronavirus infection, the winter period is characterized by the presence of seasonal diseases, which is also reflected in the significant and fairly high constant indicator for this regime.

Table 12. The effects of the regressors impact accounting for the structural breaks

	Deaths				
β_1	–0.0484	δ_1	–0.0106	μ_1	12.49
	(0.0502)		(0.239)		(8.678)
β_2	–0.0346	δ_2	–0.309*	μ_2	17.17***
	(0.0452)		(0.163)		(5.087)
β_3	0.00145	δ_3	–0.422**	μ_3	23.88***
	(0.0493)		(0.187)		23.88***
β_4	0.0115	δ_4	0.506	μ_4	39.21***
	(0.0306)		(0.446)		(7.733)
R^2	0.599				

Note. Standard errors are in parenthesis: *** — $p < 0.01$, ** — $p < 0.05$, * — $p < 0.1$. Regional fixed effects and cross-sectional averages are included in the model. The number of observations is 2890, the number of groups is 85.

Similarly, we analyze the estimation results of the model without the structural breaks, which are represented in Table 13. The results demonstrate that the model specification without structural breaks is less robust when estimating using different approaches.

Table 13. The results of the model without structural breaks

	POLS	Panel CCE	Pooled mean group
β	0.0043	–0.00394***	–0.00202
	(0.0049)	(0.00059)	(0.00282)
δ	0.206	0.00928	–0.987***
	(0.130)	(0.107)	(0.190)
μ	15.234***	2.973	—
	(2.705)	(19.60)	—
R^2	0.708		0.413

Note. Standard errors are in parenthesis: *** — $p < 0.01$, ** — $p < 0.05$, * — $p < 0.1$. Regional fixed effects and cross-sectional averages are included in the model. The number of observations is 2890, the number of groups is 85.

For the following step we extend the data period till May 14, 2023 and enlarge the dataset to 13600 observations. Similarly to the previous results, the *Udmax* test represented in Table 14 detects the structural breaks at the 5% significance level. The sequential test (Table 15) identifies 5 breaks, however the hypothesis for the absence of breaks against 1 break and 1 break against 2 breaks are rejected. Therefore, we conduct supplementary test on 0 against 3 breaks (Table 16).

Table 14. The results of the *UDmax* test — H_0 : no breaks vs. $\mathrm{H}_1 : 1 \le s \le 5$ breaks

		Bai–Perron critical values		
	Test Statistic	1% critical value	5% critical value	10% critical value
UDmax	12.30	12.37	8.88	7.46

Table 15. Sequential test for multiple breaks with unknown dates

		Bai–Perron critical values		
	Test Statistic	1% critical value	5% critical value	10% critical value
$F(1\|0)$	0.60	12.29	8.58	7.04
$F(2\|1)$	0.35	13.89	10.13	8.51
$F(3\|2)$	17.34	14.80	11.14	9.41
$F(4\|3)$	53.25	15.28	11.83	10.04
$F(5\|4)$	57.82	15.76	12.25	10.58
The number of breaks		5	5	5

Table 16. The results of the *supF*(3|0) test — H_0 : no breaks vs. H_1 : 3 breaks

		Bai–Perron critical values		
	Test Statistic	1% critical value	5% critical value	10% critical value
supF(3|0)	12.30	7.60	5.96	5.21

According to the *supF* test, we reject the null hypothesis on no breaks. The test-statistics is similar to the UDmax test, which identifies 3 breaks. Table 17 represents the detected structural breaks for the period from 2020 till 2023. The first break falls on the period from October 26, 2020 to November 02, 2020, which corresponds to the previous results. The second break is found on the week from July 19, 2021 to July 26, 2021 and the third one falls on the period from 21 till 28 February, 2022.

Table 17. The estimation of the structural break dates

Date	95% significance interval	
Week 28	Week 27	Week 29
Week 64	Week 65	Week 67
Week 95	Week 82	Week 112

Table 18 shows that all regimes are significant and have a positive sign, which confirms the increase of deaths due to COVID-19 infection. Supporting the prior estimations, the first regime has the lowest impact on the number of deaths, while the highest mortality is observed in the end of 2020 and during 2021 (second and third regimes). Our empirical findings coincides both with some recent studies (see i.e. Karpova et al., 2022) and official waves announcements.

Table 18. Estimation of the regimes

	Regime 1	Regime 2	Regime 3	Regime 4
Deaths	12.194***	39.025***	75.934***	5.659***
	(3.278)	(7.708)	(8.898)	(1.368)
Number of observations		13430		

Note. Standard errors are in parenthesis: *** — $p < 0.01$, ** — $p < 0.05$, * — $p < 0.1$.

Regime 1 coincides with the first wave of COVID-19 pandemic. Regime 2 covers the period from November 2020 to June 2021, when the second and third waves of COVID-19 spread were detected. Karpova et al. (2022) argue that these two waves can slightly be distinguished, which we indirectly confirm with our empirical findings. Regime 3 (August 2021 — February 2022) includes the 4th wave of coronavirus spread. The highest coefficient by the third regime indicates that the increase in the number of people infected by COVID-19 caused a burst of morbidity, which supports the findings of Karpova et al. (2023) and can be explained by the mutation of the virus strain. Regime 4 covers the 5th wave, which is characterized by a lighter virus strain and significant decrease in the mortality rate. Table 19 represents the detailed results of the regressors impact on the number of deaths caused by COVID-19 infection, which supports the prior findings. Interestingly, the coefficient for the increase in the number of infected people is negative and significant in the Regime 4, which implies that the spread of coronavirus did not cause the mortality increase.

Table 19. The effects of the regressors impact accounting for the structural breaks

	β_1	β_2	β_3	β_4	μ_1	μ_2	μ_3	μ_4
Deaths	0.0116***	0.0329***	0.0021*	–0.0040***	12.149***	38.624***	76.886***	6.307***
	(0.0026)	(0.0062)	(0.0011)	(0.0009)	(3.285)	(7.440)	(9.192)	(1.634)
R^2					0.76			

Note. Standard errors are in parenthesis: *** — $p < 0.01$, ** — $p < 0.05$, * — $p < 0.1$. Regional fixed effects and cross-sectional averages are included in the model. The number of observations is 13430, the number of groups is 85.

For the next step we add temperature as a control variable to our empirical setup. As in the previous case, the *UDmax* test statistics demonstrates the presence of the breaks (Table 20). According to the results of the sequential test reported in Table 21, the hypothesis of 0 breaks against 1 break and 1 break against 2 break cannot be rejected. Therefore, we test the hypothesis of no breaks against 3 breaks in order to correctly detect the number of the breaks (Table 22).

Table 20. The results of the *UDmax* test — H_0 : no breaks vs. $\mathrm{H}_1 : 1 \le s \le 5$ breaks

		Bai–Perron critical values		
	Test statistic	1% critical value	5% critical value	10% critical value
UDmax	13.32	12.37	8.88	7.46

Table 21. Sequential test for multiple breaks with unknown dates

		Bai–Perron critical values		
	Test statistic	1% critical value	5% critical value	10% critical value
$F(1\|0)$	0.56	12.29	8.58	7.04
$F(2\|1)$	0.34	13.89	10.13	8.51
$F(3\|2)$	15.27	14.80	11.14	9.41
$F(4\|3)$	89.17	15.28	11.83	10.04
$F(5\|4)$	58.00	15.76	12.25	10.58
The number of breaks			5	

Table 22. The results of the *supF*(3|0) test — H_0 : no breaks vs. H_1 : 3 breaks

		Bai–Perron critical values		
	Test statistic	1% critical value	5% critical value	10% critical value
supF(3|0)	13.32	7.60	5.96	5.21

The *supF* test statistics detects three structural breaks when controlling for the average daily temperature. Table 23 represents the dates of the breaks, which confirms the previous estimation results.

Table 23. The estimation of the structural break dates

Date	95% significance interval	
Week 28	Week 27	Week 29
Week 65	Week 64	Week 66
Week 97	Week 80	Week 114

Table 24 and 25 represent the estimation results for all regimes and the impact of regressors in case of existence of the structural breaks. Supporting the previous estimation results, the regimes significantly affect the number of deaths. The increase in the number of infected people controlling for seasonality enhances the number of deaths. The highest impact is observed for the Regime 3 and the number of infected in Regime 2.

Table 24. Estimation of the regimes

	Regime 1	Regime 2	Regime 3	Regime 4
Deaths	12.188***	38.277***	75.584***	5.677***
	(3.278)	(7.477)	(8.959)	(1.368)
Number of observations	13430			

Note. Standard errors are in parenthesis: *** — $p < 0.01$, ** — $p < 0.05$, * — $p < 0.1$.

Table 25. The effects of the regressors impact accounting for the structural breaks

	β_1	β_2	β_3	β_4	μ_1	μ_2	μ_3	μ_4
Deaths	0.0115***	0.0288***	0.0018*	–0.0042***	12.144***	38.088***	76.455***	6.317***
	(0.0026)	(0.0058)	(0.0012)	(0.0010)	(3.285)	(7.349)	(9.228)	(1.632)
R^2	0.76							

Note. Standard errors are in parenthesis: *** — $p < 0.01$, ** — $p < 0.05$, * — $p < 0.1$. Regional fixed effects and cross-sectional averages are included in the model. The number of observations is 13430, the number of groups is 85.

5. Conclusion

This paper provides an overview of various approaches for identifying structural breaks in the panel data. we describe the methods for testing a single structural break, as well as the methods for determining the multiple structural breaks. Besides, we consider approaches for identifying breaks based on regularization in addition to classical methods, which implement sequential testing. For empirical application we analyze the COVID-19 pandemic in Russian regions by estimating the impact of the increase in the number of infected people and average temperature on the mortality. Three structural breaks are found in the model, which are associated with the waves of the spread of coronavirus infection in the Russian regions.

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